$$U = A + ST.$$
$$= -kT \log Q_N + kT\left(\frac{\partial}{\partial T}[T \log Q_N]\right)_{NV}$$
$$= kT^2\left(\frac{\partial}{\partial T}\log Q_N\right)_{N,V} \quad \text{—— (7)}$$

and the specific heat at constant volume

$$C_V = \left(\frac{\partial V}{\partial T}\right)_V.$$
$$= k\left(\frac{\partial}{\partial T}\left[T^2 \frac{\partial \log Q_N}{\partial T}\right]\right)_V \quad \text{—— (8)}$$

V.V.9. <u>Thermodyniamical</u> <u>properties</u> <u>of</u> <u>an</u> <u>ideal</u> <u>gas</u>.

Let us consider an ideal gas consisting of N identical particles, each of mass 'm' enclosed in a container of volume V at a temperature T. By ideal gas we means a gas in which the interaction between the particles is negligible. So that the energy of the system is sum of the ~~km~~ kinetic energy for each particle i.e.

$$E(q,p) = \sum_{i=1}^{N} \frac{p_i^2}{2m} \quad \text{—— (1)}$$

where p_i is the momentum of the ith parti-cle. The partition function of the ideal gas can be written as,

$$Q_N = \frac{1}{N!\,h^{3N}} \iint e^{-E(q,p)/kT} \prod_{i=1}^{N} dq_i\, dp_i.$$
$$= \frac{1}{N!\,h^{3N}} \iint e^{-\sum_i p_i^2/2mkT} \prod_{i=1}^{N} dq_i\, dp_i.$$

$$= \frac{1}{N!\,h^{3N}} \iint \prod_{i=1}^{N} e^{-p_i^2/2mkT} \prod_{i=1}^{N} dq_i \, dp_i$$

$$= \frac{1}{N!\,h^{3N}} \prod_{i=1}^{N} \int e^{-p_i^2/2mkT} \, dq_i \, dp_i \quad \text{— (2)}$$

It can be rearranged as,

$$Q_N = \frac{1}{N!} [Q_1]^N \quad \text{— (3)}$$

where,

$$Q = \frac{1}{h^3} \int e^{-p_i^2/2mkT} \, dq_i \, d\vec{p}_i$$

$$= \frac{1}{h^3} \int dq_i \int e^{-p_i^2/2mkT} \, d\vec{p}_i$$

$$= \frac{V}{h^3} \int e^{-p_i^2/2mkT} \, d\vec{p}_i \quad \text{— (4)}$$

Q_1 is the partition function of a single particle using a ~~pa~~ polar co-ordinates (p_i, θ_i, ϕ_i).

$$d\vec{p}_i = p_i^2 \, dp_i \sin\theta_i \, d\theta_i \, d\phi_i.$$

(4) can be written as,

$$\cancel{Q = \frac{V}{h^3} \int_0^\infty e^{-p_i^2/2mkT} p_i^2 \, dp_i \quad \text{— (5)}}$$

$$Q = \frac{V}{h^3} \int e^{-p_i^2/2mkT} p_i^2 \, dp_i \int_0^{\pi} \sin\theta_i \, d\theta_i \int_0^{2\pi} d\phi_i$$

$$= \frac{4\pi V}{h^3} \int_0^\infty e^{-p_i^2/2mkT} p_i^2 \, dp_i \quad \text{— (5)}.$$

Let us put

$$p_i^2/2mkT = x.$$

$$p_i = \sqrt{2mkTx}$$