Kinds of Ensemble :- N, V, T, E.

① Microcanonical ensemble - N, V, E fixed T varied.
② Canonical ensemble - N, V, T fixed E varied.
③ Grand ensemble - N, T fixed N, E varied.

Monoatomic Ideal gas → Let us consider a monoatomic ideal gas consisting of N identical particles, each of mass m enclosed in a container of volume V at a temp T. The energy of a molecule may be written as,

$$\epsilon = \frac{p^2}{2m} = \epsilon_K \quad \text{— ①}$$

where the first term is the kinetic energy of the translational motion and ϵ_K is the energy associated with an internal state of the motion.

The partition function of a system in this case is given by,

$$Q_N = \frac{1}{N!}[Q_1]^N \quad \text{— ②}$$

where Q_1 is a single particle partition function.

$$Q_1 = Q_{tr} Q_{in} \quad \text{— ③}$$

Q_{tr} is the translational partition function and is given by

$$Q_{tr} = \frac{1}{h^3} \int e^{-p^2/2mkT} \, dq \, dp.$$

$$= V \left[\frac{2\pi mkT}{h^2}\right]^{3/2} \quad \text{— ④}$$

which is the equation of state for an ideal gas.

The entropy of an ideal gas is,

$$S = -\left(\frac{\partial A}{\partial T}\right)$$

$$= Nk\log\left(\frac{V}{N}\right) + \frac{3}{2}Nk\log(kT) + 3/2\,Nk + \frac{3}{2}Nk\log\left(\frac{2\pi m}{h^2}\right) + Nk.$$

$$= Nk\log\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + 5/2\,Nk.$$

or, $\frac{S}{k} = N\log\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] + 5/2\,N.$ —— (10)

which is known as the Sackur-Tetrode equation.

The energy is,

$$U = A + ST.$$

$$= 3/2\,NkT \quad \text{—— (11)}$$

and finally the specific heat at constant volume is,

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{3}{2}Nk.$$

which is independent of temperature.

—— x ——

The pressure of an ideal gas is $P = -\left(\frac{\partial A}{\partial V}\right)_{NT}$

$$= \frac{NkT}{V}$$

or, $PV = NkT.$

II nd Ensemble → Phase space of particle = configuration space + momentum space.

Volume element in phase space = $dq\,dp$.

$= dq_1 dp_1, dq_2 dp_2 \ldots dq_n dp_n.$

$\langle f \rangle = \bar{f} = \int \psi^* f \psi\, dq = \int_{-\infty}^{+\infty} p\,dq$, $\langle f \rangle = \int_{-\infty}^{+\infty} p\,dq\,dp$