Online Reading Material for Physics (H) Students (as discussed earlier)

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Q. State and prove Hamilton's principle :–

Ans:– Let us consider the three-dimensional motion of a particle of mass m in a central potential. In Cartesian coordinates the kinetic energy is

$$T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)$$

and the potential energy $V(x,y,z)$ depends only on the magnitude of $r=(x^2+y^2+z^2)^{1/2}$.

We introduce the Lagrangian

$$L = T - V = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2) - V(x,y,z) \qquad \text{——— (1)}$$

It has derivatives

$$\frac{\partial L}{\partial x} = \frac{\partial V}{\partial x} = F_x$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} = P_x$$

with similar expressions for the y and z components. The equation of motion, $F_x = \dot{P}_x$ can thus be written

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

which has the form of an Euler-Lagrange equation for the integral

$$I = \int_{t_0}^{t_1} L\,dt. \qquad \text{——— (2)}$$

This integral is called the action and Hamilton's principle of least action states that a system will evolve in such a way as to minimize the action.

The above formalism is readily extended to the case where the Lagrangian can be expressed in terms of generalized coordinates $q_i(t)$ and their time derivatives $\dot{q}_i(t)$. Then, the Euler-Lagrange equations become

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0 \qquad (i = 1, 2, 3) \text{ ——— } (3)$$

They ensure that the action is stationary with respect to arbitrary variations δq_1, δq_2 and δq_3 subject to the condition that the variations are zero at the limits of integration t_0 and t_1.

Lagrange's equations are applicable not only to classical and quantum particle dynamics but also to classical and relativistic quantum of the continuously varying space-time coordinates x_μ, and formulate Lagrange's equations in terms of a Lagrangian density $\mathcal{L}$ such that Lagrangian L is given by

$$L = \int \mathcal{L}\, d^3x .$$

This is required relation.