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For T.D.C. Part I

Paper – 2

2-dimensions geometry

SYSTEM OF CIRCLES

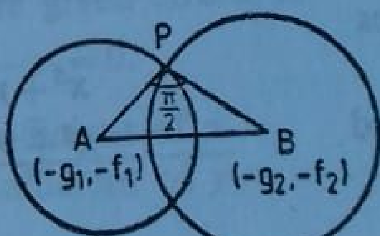
Q. ① *Find the condition that the circles*

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \text{ and } x^2+y^2+2g_2x+2f_2y+c_2=0$$

intersect orthogonally. (M. U. 1979, '81, '84, 'B. U. '85)

Solution. Let the two circles have their centres at A and B and cut at P. If the circles are orthogonal, then $AP \perp PB$.

$$\therefore \quad AB^2=AP^2+PB^2. \quad \ldots (1).$$



Now the centres of the two circles are $(-g_1, -f_1)$ and $(-g_2, -f_2)$ respectively.

Also their radii are

$$\sqrt{g_1^2+f_1^2-c_1},$$

and $\sqrt{g_2^2+f_2^2-c_2}$ respectively.

$\therefore$ The condition (1) becomes

$$(-g_1+g_2)^2+(-f_1+f_2)^2=g_1^2+f_1^2-c_1+g_2^2+f_2^2-c_2$$

or $$g_1^2+g_2^2-2g_1g_2+f_1^2+f_2^2-2f_1f_2 =g_1^2+f_1^2-c_1+g_2^2+f_2^2-c_2,$$

or $$2g_1g_2+2f_1f_2=c_1+c_2.$$

This is the required condition.

8. prove that the radical axis of two circles is perpendicular to the line joining the centres of the circles.

[M. U. 1985, '87, '90; B.U. '85; Bh. U. '86]

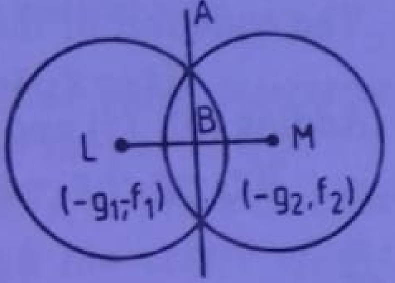
Let the two circles have for their equations

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \cdots (1)$$

and

$$x^2+y^2+2g_2x+2f_2y+c_2=0. \quad \cdots (2)$$

The equation of their radical axis AB is



$$2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0.$$

The slope (or gradient) of the radical axis $AB=-\dfrac{\text{co-efficient of } x}{\text{co-efficient of } y}$

$$=-\frac{2(g_1-g_2)}{2(f_1-f_2)}=-\frac{(g_1-g_2)}{f_1-f_2}.$$

The co-ordinates of the centres L and M of the circles (1) and (2) are respectively $(-g_1, -f_1)$ and $(-g_2, -f_2)$.

The slope of the line LM of centres

$$=\frac{\text{Difference of } y\text{-coordinates}}{\text{Difference of } x\text{-coordinates in the same order}}$$

$$=\frac{-f_2-(-f_1)}{-g_2-(-g_1)}=\frac{f_1-f_2}{g_1-g_2}.$$

$\therefore$ The slope of $AB \times$ the slope of LM

$$=-\frac{(g_1-g_2)}{f_1-f_2}\times\frac{f_1-f_2}{g_1-g_2}=-1.$$

Hence the radical axis of two circles is perpendicular to the line joining their centres.

(6)

or $$2x.0+2y.0+0=0$$

or $$0=0, \text{ which is true.}$$

Hence the three radical axes of the three circles, taken in pairs, meet in a point.

The point is called the radical centre of the three circles.

The radical axes of three circles, taken in pairs, are concurrent.

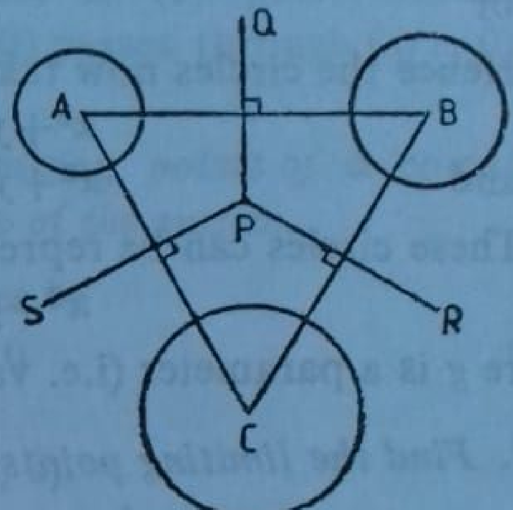
(R. U. 1967; P. U. '68)

Solution. Let the three circles be

$$x^2+y^2+2g_1x+2f_1y+c_1=0$$

$$x^2+y^2+2g_2x+2f_2y+c_2=0$$

and $x^2+y^2+2g_3x+2f_3y+c_3=0.$



Radical axes of (1), (2); (2), (3) and (3), (1) are respectively

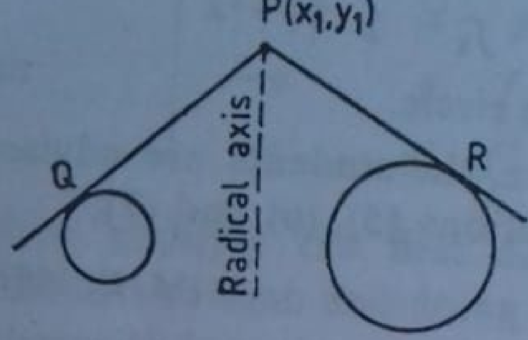
$$2x(g_1-g_2)+2y(f_1-f_2)+c_1-c_2=0 \quad .. \quad (4)$$

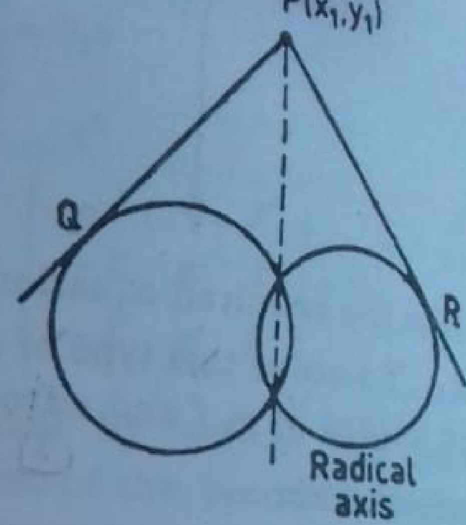
$$2x(g_2-g_3)+2y(f_2-f_3)+c_2-c_3=0. \quad .. \quad (5)$$

and $$2x(g_3-g_1)+2y(f_3-f_1)+c_3-c_1=0. \quad .. \quad (6)$$

Adding (4), (5) and (6) we get

$$2x(g_1-g_2+g_2-g_3+g_3-g_1)+2y(f_1-f_2+f_2-f_3+f_3-f_1)+c_1-c_2+c_2-c_3+c_3-c_1=0$$





Then $$PQ=\sqrt{x_1^2+y_1^2+2g_1x_1+2f_1y_1+c_1}$$

and $$PR=\sqrt{x_1^2+y_1^2+2g_2x_1+2f_2y_1+c_2}.$$

We have $$PQ=PR$$

or $$PQ^2=PR^2$$

or $$x_1^2+y_1^2+2g_1x_1+2f_1y_1+c_1=x_1^2+y_1^2+2g_2x_1+2f_2y_1+c_2$$

or $$2(g_1-g_2)x_1+2(f_1-f_2)y_1+c_1-c_2=0.$$

Hence the locus of (x_1, y_1) is

$$2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0.$$

This is the required equation of the radical axis. Evidently represents a straight line, as it is of the first degree in x and y.