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3) State and prove reciprocity theorem.

Ans:- Statement:. In any branch of a network, the current (I) due to a single source of voltage (V) elsewhere in the network is equal to the current through the branch in which the source was originally placed when the source is placed in the branch in which the current (I) was originally obtained.

or,

In any linear network, if a source of e.m.f. E located in one mesh, produces a current I_2 in second mesh, then if the same source E is placed in the second mesh and it produces a current I_1' in first mesh, we have $I_2 = I_1'$.

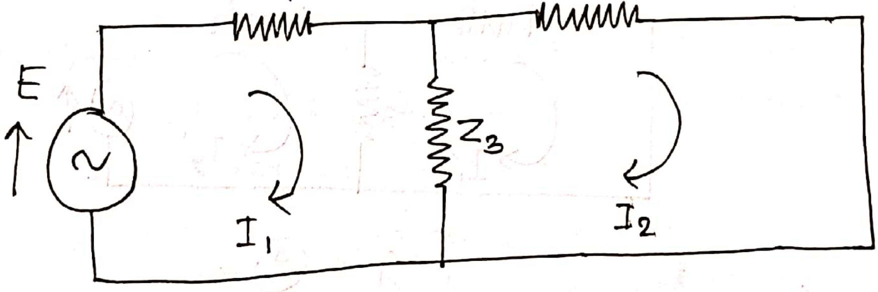

Fig - 1.

Explanation:- Let us consider the circuit shown in fig-(1). Let us calculate the value of (E/I_2) with the help of Kirchhoff's Law.

Applying this law, we have

$$E = (Z_1 + Z_3) I_1 - Z_3 I_2 \quad \text{——— (1)}$$

$$0 = -Z_3 I_1 + (Z_2 + Z_3) I_2 \quad \text{——— (2)}$$

Solving the eqns. (1) and (2), we get

$$I_2 = \frac{\begin{vmatrix} Z_1 + Z_3 & E \\ -Z_3 & 0 \end{vmatrix}}{\begin{vmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2+Z_3 \end{vmatrix}}$$

$$= \frac{Z_3 E}{(Z_1+Z_3)(Z_2+Z_3) - Z_3^2}$$

$$= \frac{Z_3 E}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}$$

$$\therefore \frac{E}{I_2} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3} \quad \text{——— (3)}$$

Now let the position of the battery be changed and the circuit be as shown in fig - 2.

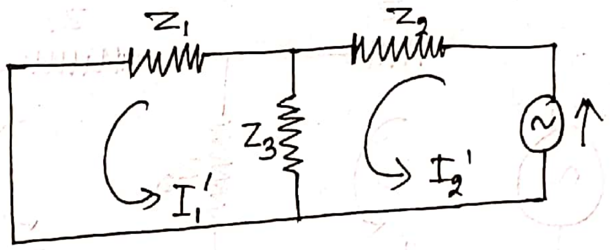

Fig - (2).

Applying Kirchhoff's Law, we have.

$$E = (Z_2 + Z_3) I_2' - Z_3 I_1' \quad \text{——— (4)}$$

$$0 = -Z_3 I_2' + (Z_1 + Z_3) I_1' \quad \text{——— (5)}$$

solving eqn (4) & (5), we get.

$$I_1' = \frac{\begin{vmatrix} (Z_2+Z_3) & E \\ -Z_3 & 0 \end{vmatrix}}{\begin{vmatrix} Z_2+Z_3 & E \\ -Z_3 & Z_1+Z_3 \end{vmatrix}}$$

$$= \frac{Z_3 E}{(Z_2+Z_3)(Z_1+Z_3) - Z_3^2}$$

$$= \frac{Z_3 E}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}$$

$$\therefore \frac{E}{I_1'} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3} \quad \text{—— (6)}$$

As the right hand side of eqns. (3) & (6) are equal, hence

$$(E/I_2) = (E/I_1')$$

This proves the reciprocity theorem.

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