⇒ **Boolean Algebra :-**

The ckt. in digital computers follow the logic of minds. This logic is binary (0 & 1) and resembles ordinary algebra called Boolean Algebra. These are three operations:-

(i) OR addition represented by a (+) sign.

(ii) AND multiplication represented by (X) or (•) sign.

(iii) NOT operation is represented by a bar over a variable.

OR Addition.

$Y = A + B$.

for n- inputs

$Y = A + B + C + \cdots$
$\cdots + n$.

Rules. ↓

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

AND multiplication:

$$Y = A \cdot B$$

for n-inputs

$$Y = A \cdot B \cdot C \cdot \ldots N$$

Rules:

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$

NOT operation:-

$$Y = \overline{A} = A'$$

Rules

$\overline{0} = 1$
$\overline{1} = 0$

⇒ De-Morgan's theorem:-

Theorem I:- The complement of the sum of two or more variables is equal to the product of the complements of the variables.

If A & B are two variables.

$$\overline{A+B} = \overline{A} \cdot \overline{B} \quad \text{——— (1)}$$

Theorem II:- The complement of the product of two or more variables is equal to the sum of the complements of the variables.

$$\overline{A \cdot B} = \overline{A} + \overline{B} \quad \text{——— (2)}$$

→ To prove $\overline{A+B} = \overline{A} \cdot \overline{B}$

(i) when $A = 0$, $B = 0$.

$\overline{A+B} = \overline{0+0} = \overline{0} = 1$

and $\overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{0} = 1 \cdot 1 = 1$

Hence. $\overline{A+B} = \overline{A} \cdot \overline{B}$.

(ii) when $A = 0$, $B = 1$, $\overline{A+B} = \overline{0+1} = \overline{1} = 0$

and $\overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{1} = 1 \cdot 0 = 0$

Hence. $\overline{A+B} = \overline{A} \cdot \overline{B}$.

(iii) when $A = 1$, $B = 0$, $\overline{A+B} = \overline{1+0} = \overline{1} = 0$.

and $\overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{0} = 0 \cdot 1 = 0$.

Hence, $\overline{A+B} = \overline{A} \cdot \overline{B}$

(iv) when $A = 1$, $B = 1$, $\overline{A+B} = \overline{1+1} = \overline{1} = 0$.

and $\overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{1} = 0 \cdot 0 = 0$ Hence. $\overline{A+B} = \overline{A} \cdot \overline{B}$

→ To prove $\overline{A\cdot B} = \overline{A}+\overline{B}$

(i) When $A=0$, $B=0$. $\overline{A\cdot B} = \overline{0\cdot 0} = \overline{0} = 1$

and $\overline{A}+\overline{B} = \overline{0}+\overline{0} = 1+1 = 1$

Hence. $\overline{A\cdot B} = \overline{A}+\overline{B}$.

(ii) When, $A=0$. $B=1$.

$\overline{A\cdot B} = \overline{0\cdot 1} = \overline{0} = 1$

and $\overline{A}+\overline{B} = \overline{0}+\overline{1} = 1+0 = 1$

Hence. $\overline{A\cdot B} = \overline{A}+\overline{B}$.

(iii) When, $A=1$, $B=0$. $\overline{A\cdot B} = \overline{1\cdot 0} = \overline{0} = 1$

and $\overline{A}+\overline{B} = \overline{1}+\overline{0} = 0+1 = 1$

Hence. $\overline{A}\cdot\overline{B} = \overline{A}+\overline{B}$

(iv) When. $A=1$ & $B=1$

$\overline{A\cdot B} = \overline{1\cdot 1} = \overline{1} = 0$

and $\overline{A}+\overline{B} = \overline{1}+\overline{1} = 0+0 = 0$.

Hence, $\overline{A\cdot B} = \overline{A}+\overline{B}$.

Thus De-morgon's theorem proved.