# RUNGE-KUTTA METHOD

## (R-K METHOD)

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M.Sc. Sem II
Differential Eqn.
UNIT-3

To achieve better accuracy, Runge and Kutta jointly proposed a method called the fourth-order method for solving the differential equation.

Let us suppose we have differential equation

$$\frac{dy}{dx} = f(x, y) \quad \text{——} \quad (1)$$

with the initial condition: $y = y_0$ at $x = x_0$.
The $f(x, y)$ is a function of $x$ and $y$.

According to Runge-Kutta (R.K) method, solution of the first order linear differential eqn. (1) is.

$$y_{i+1} = y_i + \frac{h}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right) \quad \text{——} \quad (2)$$

Where, $K_1$, $K_2$, $K_3$ and $K_4$ are

$$K_1 = f(x_i, y_i)$$

$$K_2 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_1\right)$$

$$K_3 = f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_2\right)$$

$$K_4 = f(x_i + h, y_i + hK_3)$$

FORTRAN program for solving the first order, first degree linear differential equation by using the equation (2) is RUNGE-KUTTA METHOD FIRST, where the initial values of $x$ and $y$ are X1 and Y1, respectively, and the final value of $x$ is XN. N is an integer which is taken to be a large number for better accuracy.

values of X1, Y1, XN, N, and the content of the function subprogram FUNCTION F(X,Y), which is $f(x,y)$, are to be incorporated.

* RUNGE-KUTTA METHOD FIRST

```
      OPEN (UNIT=2, FILE='OUTPUT')
      X1 =
      Y1 =
      XN =
      N =
      H = (XN - X1)/N      or,  XN-X1/N
      H2 = H/2
10    CONTINUE
      AK1 = F(X1, Y1)
      AK2 = F(X1+H2, Y1+H2* AK1)
      AK3 = F(X1+H2, Y1+H2*AK2)
      AK4 = F(X1+H, Y1+H* AK3)
      Y1 = Y1 + H*(AK1+ 2*AK2+ 2*AK3+AK4)/6
      X1 = X1+H
      IF(X1.LT.XN) GO TO 10
      WRITE (2, 20) X1, Y1
20    FORMAT (2X, 'X=', F5.1,3X, 'Y=', F9.5)
      STOP
      END
      FUNCTION F(X,Y)
```

——— Write down FORTRAN statements
——— for the expression of the
——— function $f(x, y)$

```
RETURN
END
```

—— × —— KRB

Exercise: For the differential equation

$$\frac{dy}{dx} = \frac{2x+5}{2y}$$

We have $y = 2$ at $x = 1$. Find out the value of $y$ at $x = 4$ with the help of the Runge Kutta method.

Ans: For better accuracy, value of N is taken reasonably large. Let us take it, say, 500. Thus in the program RUNGE-KUTTA METHOD FIRST, the input values are

```
X1 = 1
Y1 = 2
XN = 4
N = 500
```

For $f(x, y)$, the Function subprogram is

```
FUNCTION F(X, Y)
  F = (2*X +5)/(2*Y)
RETURN
END
```

The output of the program (Content of the file OUTPUT) is

X = 4.0   Y = 5.83096

BASIC program for the Exercise 4 is

```
10  REM RUNGE-KUTTA METHOD FIRST
20  DEF FNA(X,Y) = (2*X+5)/(2*Y)
30  X1 = 1
40  Y1 = 2
50  XN = 4
60  N = 500
70  H = (XN-X1)/N
80  H2 = H/2
90  AK1 = FNA(X1, Y1)
100 AK2 = FNA(X1+H2, Y1+H2*AK1)
110 AK3 = FNA(X1+H2, Y1+H2*AK2)
120 AK4 = FNA(X1+H, Y1+H*AK3)
130 Y1 = Y1+H*(AK1+2*AK2+2*AK3+
                AK4)/6
140 X1 = X1+H
150 IF (X1<XN) THEN 90
160 PRINT "X="; X1, "Y="; Y1
170 END
```