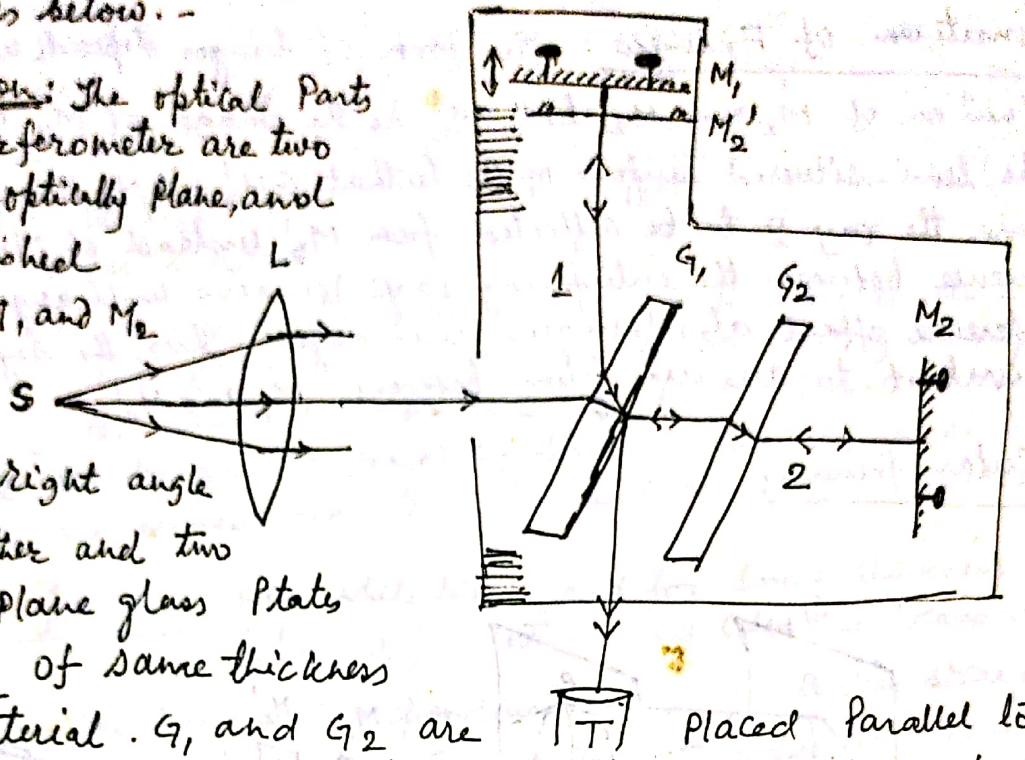
Michelson Interferometer

An interferometer is an apparatus devised to produce interference fringes and to measure different characteristics of interference. The interferometer developed by Michelson has been shown by schematic diagram as below.-

Construction: The optical Parts of the interferometer are two excellant optically Plane, and highly Polished Mirrors M_1 and M_2 Placed at right angle to each other and two optically Plane glass Plates G_1 and G_2 of same thickness and material. G_1 and G_2 are Placed Parallel to each other and at an angle of 45^0 with M_1 and M_2. The surface of G_1 towards G_2 is semisilvered. M_1 is mounted on a Carriage and can be moved in the direction of arrows along an accurately machined track. Its motion is controlled by a very fine micrometer screw. The Mirrors M_1 and M_2 carry adjusting screws at their back by which they can be tilted about horizontal and vertical axes.



Working: Light from an extended source S is ~~made~~ made nearly Parallel by a lens L enters G_1. At the semisilvered surface of G_1, incident ray is Partially reflected towards Mirror ~~on~~ M_1 and rest is transmitted towards Mirror M_2. After reflection at M_1 and M_2, the rays combine at the semi-silvered surface and enter a telescope T. The rays entering the telescope have been obtained from the same source and hence they have a constant Phase difference between them and are in a Position to produce sustained interference.

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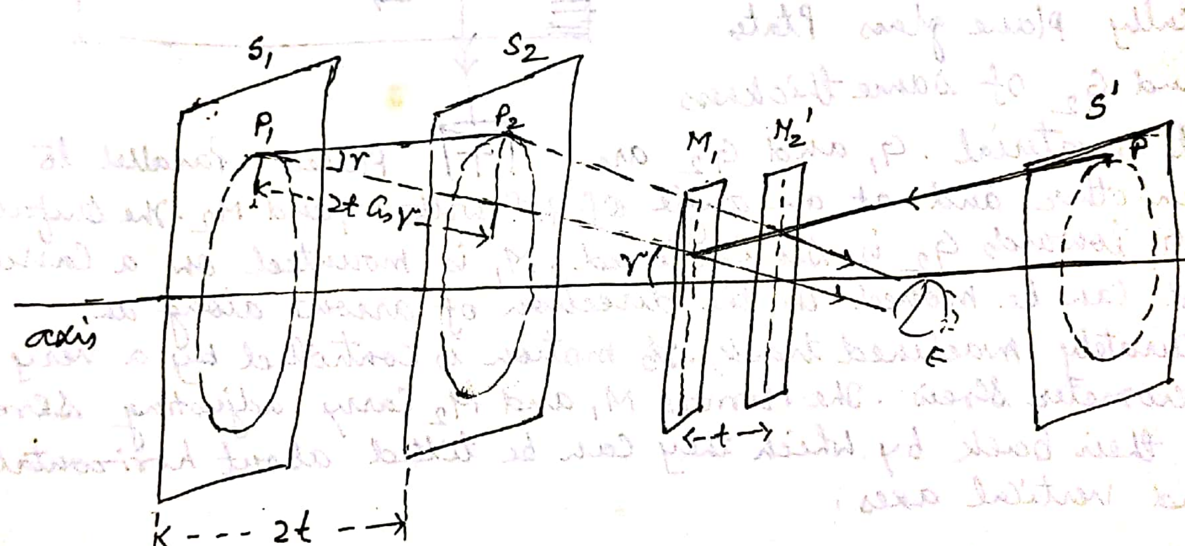
In absence of G_2, ray no. 1 travels through glass twice, while ray no. 2 does not do so even once. Thus in absence of G_2 the paths of rays 1 and 2 in glass are not equal. To equalise these paths, Glass plate G_2 of same thickness and material is placed parallel to G_1. G_2 is therefore called Compensating Plate.

Formation of Fringes: The form of fringes depends upon the inclination of M_1 and M_2. Let M_2' be the image of M_2 due to reflection at the semi-silvered surface of G_1 so that $G_1M_2' = G_1M_2$. Therefore if we assume the ray 2 to be reflected from M_2' instead of M_2, the path difference between the interfering rays remains unchanged, hence the interference effect also remain unchanged. Thus, the system is equivalent to an air film between M_1 and M_2'.

Circular fringes:



When M_2 is exactly perpendicular to M_1 the film M_1M_2' is of uniform thickness and we obtain circular fringes localised at infinity explained as below.

M_1 and M_2' are parallel reflectors. The actual source is replaced by a virtual source S' formed by reflection at semi silvered surface of G_1. S' forms two virtual images S_1 and S_2 in mirrors M_1 and M_2' respectively. The light from a point P on the extended source appears to come from P_1 & P_2 on S_1 & S_2 respectively. If $t = M_1M_2'$ then $2t = S_1S_2$. The path difference between the rays entering the eyes is $2t\cos r$.

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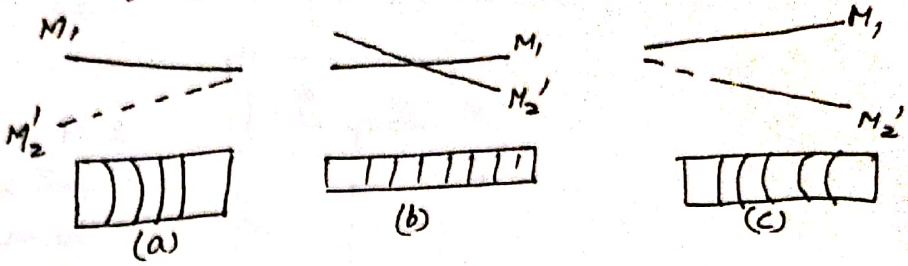
Now P appears bright, if $2t\cos r = n\lambda$

and P appears dark if $2t\cos r = (2n+1)\lambda/2$

The locus of the Point of the source making same angle at the axis is a circle. Hence, a series of bright and dark circular fringes is seen. Fringes are at infinity because interfering rays are Parallel.

<u>Localised fringes</u>



When M_2 is not Perpendicular to M_1, the air film between M_1 & M_2' is wedge shaped and fringes are of nature shown in (a) & (c). As the light incident on the film at different angles, Curved fringes with convexity towards the thin edge of the wedge are obtained. If the thickness of the film is very small, the fringes are practically straight as shown in fig (b). These fringes are formed near the film and are called localised fringes.

<u>White Light fringes:</u> With white light and for small thickness of the film, a few curved and coloured localised fringes are obtained, the fringes corresponding to $t=0$ is perfectly straight and achromatic. For large thickness of the film uniform illumination is obtained.

Determination of wavelength:

The interferometer is adjusted to obtain circular fringes in the telescope. The Position of M_1 is now so adjusted that the bright spot appears at centre of field of view. If t be the thickness of the film and n be the order of spot, we have $2t\cos r = n\lambda$

At the Centre $r=0$, so $\cos r = 1$ and hence $2t = n\lambda$ — (1)

If now M_1 is moved away from M_2' by $\lambda/2$ then $2t$ changes by λ and $(n+1)$ replaces n in eqn. (1) Hence $(n+1)$th bright spot appear at the centre. Thus each time M_1 is moved through a distance of $\lambda/2$, next bright spot appear at the centre of the field. If during the movement of M_1 displacement is D for N new fringes appearing at the Centre. Then $D = N \cdot \lambda/2$ so $\lambda = 2D/N$. The displacement is measured with the micrometer screw and N is counted visually. Hence λ can be calculated.

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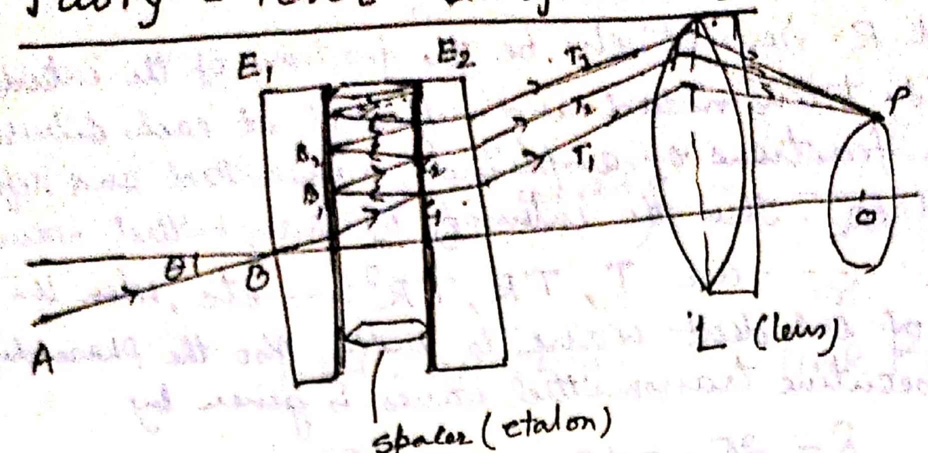
Fabry - Perot Interferometer



<u>Construction</u>: A Fabry - Perot interferometer consists of two glass plates forming a parallel air film between them. The faces of two plates in front of each other which have a parallel air film between them are lightly silvered. The outer surfaces are plane and wedge shaped. It is done so to avoid interference between the rays reflected at the outer unsilvered surface. One plate is fixed and other is attached to a carriage which can be moved towards or away from fixed plate and the shift can be measured accurately by scale.

<u>Principle</u>: The instrument works on the Principle of interference produced by multiple reflection in the air film between two plates lightly silvered.

<u>working</u>: As shown in the above fig, Let a monochromatic light from a broad source be incident on the plate E_1 at an angle θ. As a result of multiple internal reflections and consequent transmission we obtain a set of infinite parallel transmitted waves like C_1T_1, C_2T_2 ... etc. Since these waves are obtained from the same incident wave, they interfere when combined in the 2nd focal plane of lens L.

The interference fringes are circular in nature. The condition for maxima is $2d\cos\theta = n\lambda$
where d = spacing between parallel plates
& n = order of fringes.

The interference pattern consists of a system of bright concentric rings on a dark background, each ring corresponds to a particular value of θ.

(2)

Intensity distribution:

Let T and R respectively be the fractions of the incident light intensity transmitted and reflected at each silvered surface; The fractions of amplitude transmitted and reflected are $\sqrt{T}$ and $\sqrt{R}$. Then the intensity of transmitted waves C_1T_1, C_2T_2, ... are T, TR, TR^2... etc, when the amplitude of incident wave is unity. Also the phase difference between consecutive transmitted waves is given by

$$\delta = \frac{2\pi}{\lambda} \times 2d\cos\theta \qquad \text{—— (11)}$$

Therefore, if the incident wave is represented by $y = \sin\omega t$, the successive transmitted waves can be represented by $y_1 = T\sin\omega t$, $y_2 = TR\sin(\omega t - \delta)$

$y_3 = TR^2\sin(\omega t - 2\delta)$ ... and so on.

The negative signs are there because phase angles decreases as the path difference increases.

The final resultant vibration at P can be written as $D\sin(\omega t - \phi)$ where D is the instaneous resultant amplitude and ϕ is the resultant phase. Now by principle of superposition of wave motions, we have

$$D\sin(\omega t - \phi) = T\sin\omega t + TR\sin(\omega t - \delta) + TR^2\sin(\omega t - 2\delta) + \cdots$$

Expanding sine terms and equating the co-efficients of $\sin\omega t$ and $\cos\omega t$ on the two sides we may obtain

$$D\cos\phi = T + TR\cos\delta + TR^2\cos 2\delta + \cdots$$

and $$D\sin\phi = TR\sin\delta + TR^2\sin 2\delta + TR^3\sin 3\delta + \cdots$$

Hence the resultant intensity (I) is given by

$$I = D^2 = (D\cos\phi + iD\sin\phi)(D\cos\phi - iD\sin\phi)$$

where $i = \sqrt{-1}$

Here $$(D\cos\phi + iD\sin\phi) = T(1 + Re^{i\delta} + R^2e^{2i\delta} + \cdots)$$

$$= \left(\frac{T}{1 - Re^{i\delta}}\right)$$

and $\left(D\cos\phi - iD\sin\phi\right) = T\left(1+Re^{-i\delta} + R^2e^{-2i\delta} + \cdots\right)$

$$= \frac{T}{1-Re^{-i\delta}}$$

Hence $$I = \frac{T^2}{(1-Re^{i\delta})(1-Re^{-i\delta})} = \frac{T^2}{1+R^2-2R\cos\delta}$$

$$= \frac{T^2}{(1-R)^2+2R-2R\cos\delta} = \frac{T^2}{(1-R)^2+4R\sin^2\delta/2}$$

$$= \frac{T^2}{(1-R)^2}\left[\frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\delta/2}\right] \quad \text{---- (III)}$$

This expression gives the variation of resultant intensity with $\sin^2\delta/2$ and with the film properties.

For maximum intensity, $\sin^2\frac{\delta}{2}=0$ or $\delta = 2n\pi$

$$\therefore I_{max} = \frac{T^2}{(1-R)^2}$$

For minimum intensity, $\sin^2\delta/2 = 1$ or $\delta=(2n+1)\pi$

$$\therefore I_{min} = \frac{T^2}{(1+R)^2}$$

when there is no absorption at each reflecting surface

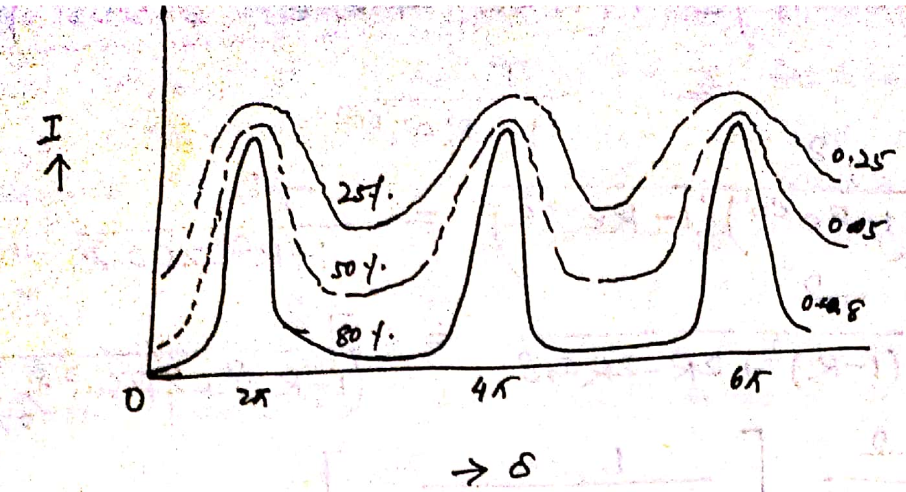
$$T+R=1 \quad \text{or} \quad T=(1-R)$$

and thus $I_{max} = \frac{T^2}{(1-R)^2} = 1$ and $I_{min} = \frac{(1-R)^2}{(1+R)^2}$

In general equation (III) can be written as

$$I = I_{max}\left\{\frac{1}{1+F\sin^2\delta/2}\right\} \quad \text{where } F = \frac{4R}{(1-R)^2}$$

<u>sharpness of fringes</u>: The plot of I verses δ shows that intensity depends upon R, the reflectivity. When R is very ~~char~~ large ~~the quantity~~ the intensity falls more rapidly on either side of Maximum and greater is the difference between Max^m & Min^m.



Further the visibility of the fringes is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{I_{max}}{I_{min}} - 1 \Big/ \frac{I_{max}}{I_{min}} + 1$$

$$= \frac{(1+R)^2}{(1-R)^2} - 1 \Big/ \frac{(1+R)^2}{(1-R)^2} + 1 = \frac{2R}{(1+R^2)}$$

Thus it follows that visibility of the fringes depend upon only the reflecting power of the Surfaces and is independent of transmission Co-efficient.

Determination of wavelength

The interferometer is first adjusted to form Circular fringes in the Centre of the field of view. Suppose the seperation between the plates is such that a bright fringe of order n is obtained at the Centre. In this Condition

$$2d = n\lambda$$

It is clear that each time the movable plate moves through a distance $\lambda/2$ and the next bright fringe Comes into field of view.

To determine λ, the movable plate is moved from the position x_1 to a position x_2 and the no. of bright fringes N that appears at the Centre are Counted. Then evidently

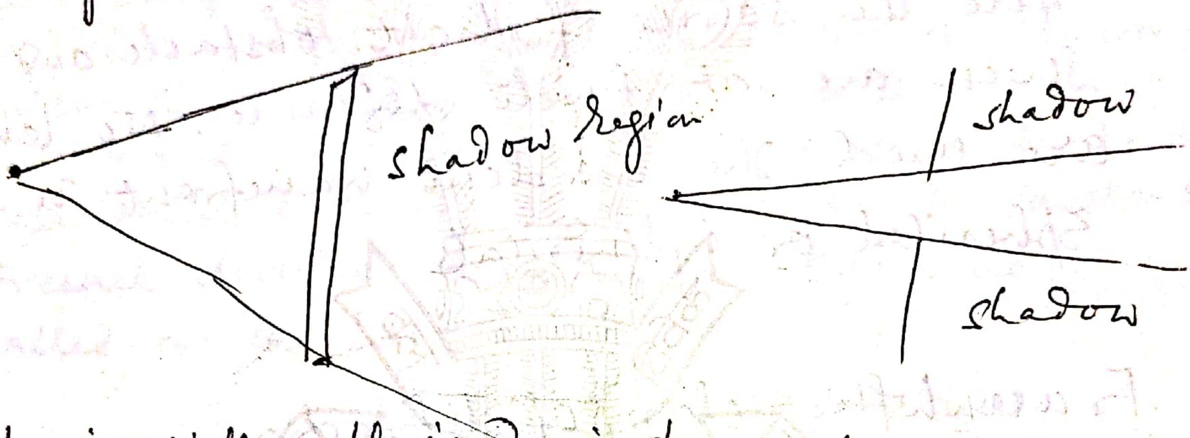
$$N \cdot \lambda/2 = x_2 - x_1$$

$$\therefore \lambda = \frac{2(x_2 - x_1)}{N}$$, From this relation λ is Computed.

Q. What is diffraction? Distinguish between Freshel diffraction and Fraunhoffer diffraction. Explain Freshel's half period zone theory of diffraction.

Deffraction.

It is a matter of common experience that there is a shadow region behind an obstacle or if a light passes through a slit, there is also shadow region below & above the aperture.



It is well explained in terms of rectilinear propagation of light according to Corpuscular theory. When the size of the obstacle or hole is made smaller (of the order of wavelength of light), some enchroachment of light in the shadow region was observed. It is due to bending of light round the corner of an obstacle in the shadow region and is called diffraction of light. Actually some fringe pattern was observed in the shadow region due to bending. This was later on explained by wave theory of light. We may call diffraction as the special case of interference.

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The phenomenon of diffraction was studied first by Fresnel & then by Fraunhoffer. So it is of two kinds: Fresnel class of diffraction and Fraunhoffer class of diffraction.

Fresnel class of diffraction —

Here the source of light, obstacle and the screen are at finite distance. No lenses are used. The incident wavefront is either spherical or cylindrical

Fraunhoffer class of diffraction —

Here the source of light, obstacle (slits) and the screen are infinite distances. This is done by placing the source and the screen in the focal planes of two lenses. The incident wavefront is plane. The diffraction pattern must be observed in the plane which is conjugate to the plane in which source of light lies.

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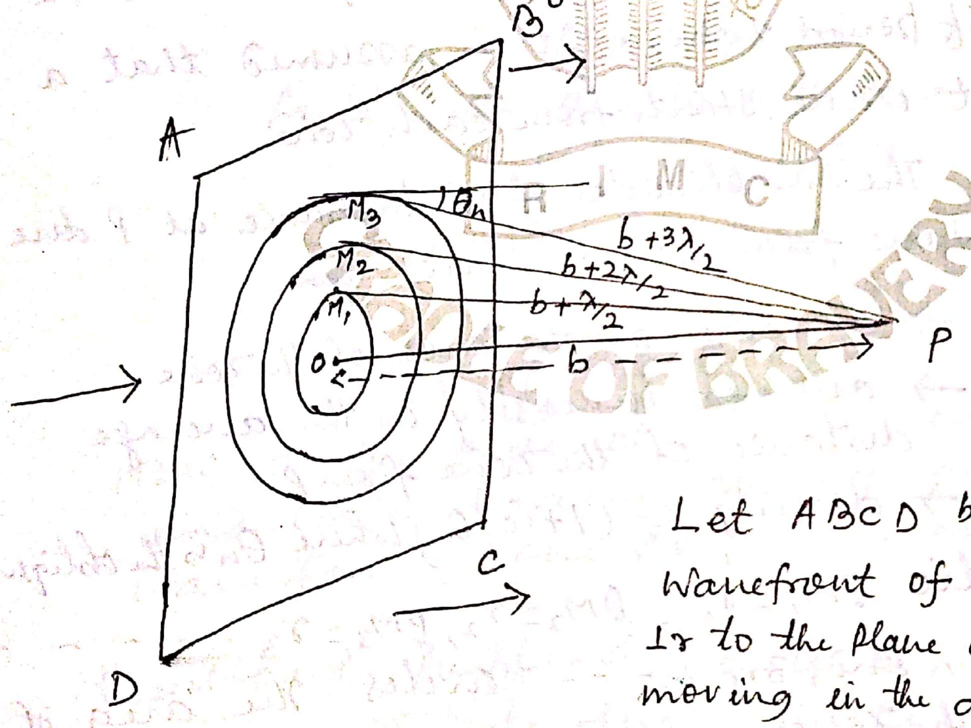
Fresnel's Half Period zones

According to wave theory of light, each Point in a light source sends out waves in all directions. The Continuous locus of Particles vibrating in same Phase is called the wave front. Each point of the wavefront is the source of Secondary wavelets. Fresnel assumed that these wavelets are in a position to interfere and that the resultant intensity of light at any Point is the result of interference of these wavelets. In order to find out the resultant intensity at a point due to wavefront, Fresnel divided the wavefront into a no. of zones called as the half period zones



Let ABCD be a plane wavefront of wavelength λ $\perp$r to the Plane of the Paper & moving in the direction from O to P

Let P be an external Point at which the ~~entire~~ effect of the entire wavefront is to be found.

In order to achieve this, Fresnel sub-divided the wavefront into a no. of half-period zones as follows.

Let us draw a $\perp r$ PO on the wavefront from P. The point O is the pole of the wavefront w.r.to P. Let $PO = b$, with P as centre and radii: $PM_1 = b + \lambda/2$, $PM_2 = b + 2\lambda/2$, $PM_3 = b + \frac{3\lambda}{2}$ ... and so on draw a series of spheres. The sections of these spheres by the plane of the wavefront are concentric circles with O as common centre. The area of the first circle is the first half period zone. The area between 2nd & first circle is the second half period zone and so on. The secondary wavelets originating from O & M_1 and reaching at P will have a path diff. of $\lambda/2$ or a time diff. of half period. It is why the zones are called half period zones. It is assumed that a resultant wave starts from each zone.

The amplitude of disturbance at P due to the wave from a zone varies ~~as~~

→ directly as the area of the zone

→ ~~average~~ inversely as the average distance of the zone from P

→ directly as $(1+\cos\theta_n)$ where θ_n is the obliquity.

Let radii $OM_1 = r_1$, $OM_2 = r_2$, $OM_3 = r_3$ ..... of first, 2nd, 3rd ... circles. The area of the nth zone

$$= \pi r_n^2 - \pi r_{n-1}^2$$

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$$= \pi\left[\left\{\left(b+\frac{n\lambda}{2}\right)^2 - b^2\right\} - \left\{\left(b+(n-1)\lambda/2\right)^2 - b^2\right\}\right]$$

$$= \pi\left[\cancel{\theta}\, b n\lambda + \frac{n^2\lambda^2}{4} - b(n-1)\lambda - (n-1)^2\frac{\lambda^2}{4}\right]$$

$$= \pi\left[b\lambda + \frac{\lambda^2}{4}\left\{n^2-(n-1)^2\right\}\cancel{\ }\right]$$

$$= \pi\left[b\lambda + \frac{\lambda^2}{4}(2n-1)\right]$$

$$= \pi b\lambda \text{ approx.}$$

because b is quite large in comparison to λ so λ^2 is neglected. In this way the area of each zone is approximately the same

Further the average distance of nth zone from P is $\dfrac{\left[b+\frac{n\lambda}{2} + b + (n-1)\lambda/2\right]}{2} = b+(2n-1)\lambda/4$

$\therefore$ Amplitude due to nth zone

$$\propto \frac{\pi\left[b\lambda + (2n-1)\lambda^2/4\right]}{b+(2n-1)\lambda/4}(1+\cos\theta_n)$$

$$\propto \pi\lambda(1+\cos\theta_n)$$

Now as the order of the zone increases, θ_n increases and $\cos\theta_n$ decreases. Thus; the amplitude of the wave from a zone at P decreases as n increases.

Resultant Amplitude — If $R_1, R_2, R_3 \ldots$ be the amplitude due to the 1st, 2nd, 3rd ... zones respectively at P, then the resultant amplitude is given by

$$R = R_1 + R_2 + R_3 + \cdots + R_n$$

if n is odd (if n is even, the last term is $-R_n$)

Now R_1 is slightly greater than R_2 & R_2 than R_3 and so on, they may be taken to form an arithmetic progression and hence we may write

$$R_2 = \frac{R_1 + R_3}{2}, \quad R_4 = \frac{R_3 + R_5}{2} \quad \text{--- and so on}$$

$$\therefore R = \frac{R_1}{2} + \left[\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right] + \left[\frac{R_3}{2} - R_4 + \frac{R_5}{2}\right] + \cdots$$

The various terms within brackets reduce to zero because $R_2 = \frac{R_1}{2} + \frac{R_3}{2}$ & $R_4 = \frac{R_3}{2} + \frac{R_5}{2}$ . . .

$$\therefore R = \frac{R_1}{2} + \frac{R_n}{2}, \text{ if } n \text{ is odd}$$

and $R = \frac{R_1}{2} + \frac{R_{n-1}}{2} - R_n$ if n is even

If n is quite large, R_n and R_{n-1} are almost ineffective on account of the distance and obliquity factor. Hence resultant amplitude at P due to the whole wave ~~front~~, whether n is odd or even, is

$$R = R_1/2$$

Thus, the resultant amplitude is only one half of that which would be produced by the first half period zone.

As the intensity $I \propto (\text{Amplitude})^2$

therefore $I = R_1^2/4$

Thus the intensity at P due to the whole wavefront is only one fourth of that due to the first half period zone alone.

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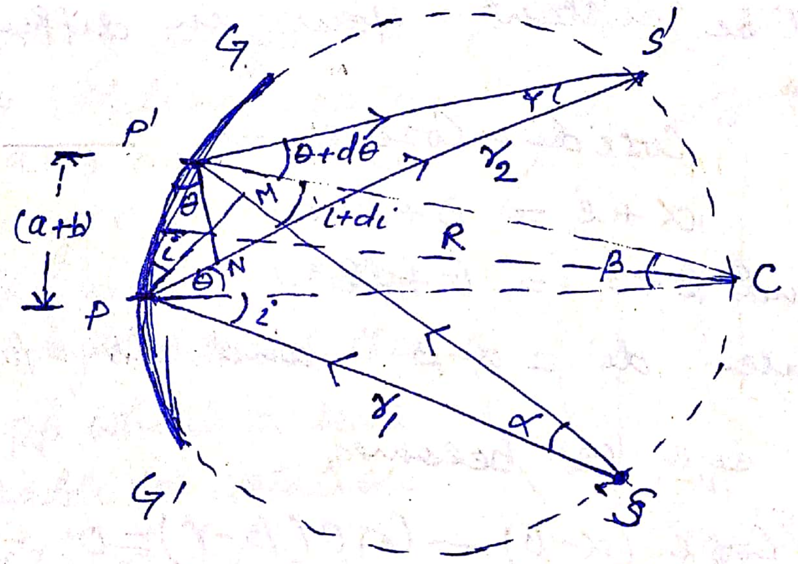
Theory of Concave grating & Eagle's Mounting

Concave grating – If a polished spherical surface of metal, like speculum, is ruled with parallel lines equally spaced along the chord of its arc, it diffracts light and brings the diffracted rays to focus without any converging lens. Such an arrangement is known as concave grating.

Theory of Concave grating



Let GG' be the surface of a concave grating with C as the centre of curvature and R be the radius of curvature. P and P' be the two consecutive points such that $PP' = (a+b)$ is the grating element. S is a linear source of light on the circumference of the dotted circle sending light of wavelength λ. Let SP and SP' be the incident wave and PS' and P'S' the corresponding diffracted waves. Let angle of incidence be

i and $i+di$ respectively and corresponding angle of diffraction θ & $\theta+d\theta$ respectively.

The path difference between rays SPS' and $SP'S'$

$$= (SP' + P'S') - (SP + PS')$$
$$= (SP' - SP) - (PS' - P'S')$$
$$= P'M - PN = PP' \sin i - PP' \sin\theta$$
$$= PP'(\sin i - \sin\theta) = (a+b)(\sin i - \sin\theta)$$

For the rays to produce maximum at s', we should have

$$(a+b)(\sin i - \sin\theta) = n\lambda \quad \text{—— (1)}$$

In order that all diffracted rays of a given wavelength may come to focus at s', the path difference $(a+b)(\sin i - \sin\theta)$ should be constant. Hence on differentiating eqn (1), we get

$$\cos i \, di - \cos\theta \, d\theta = 0 \quad \text{—— (2)}$$

Now $\alpha + i = \beta + i + di$

and $\beta + \theta = \theta + d\theta + \gamma$

whence $di = \alpha - \beta$ and $d\theta = \beta - \gamma$

Hence eqn. (2) becomes

$$\cos i(\alpha - \beta) - \cos\theta(\beta - \gamma) = 0 \quad \text{—— (3)}$$

If $PS = r_1$, $PS' = r_2$ and R = radius of curvature

then $r_1 \cdot \alpha = PM = (a+b)\cos i$

$R \cdot \beta = PP' = (a+b)$

$r_2 \cdot \gamma = P'N = (a+b)\cos\theta$

$$\therefore \alpha = \frac{(a+b)\cos i}{r_1}, \quad \beta = \frac{a+b}{R}, \quad \gamma = \frac{(a+b)\cos\theta}{r_2}$$

Putting these values of α, β and γ in eq. (3) and dropping the Common factor $(a+b)$, we get

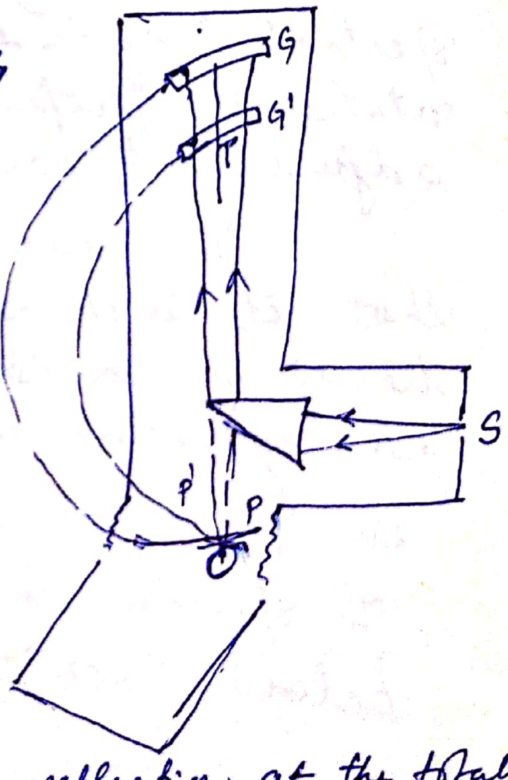
$$\cos i\left[\frac{\cos i}{r_1}-\frac{1}{R}\right]-\cos\theta\left[\frac{1}{R}-\frac{\cos\theta}{r_2}\right]=0$$

If this equation is to be satisfied for all values of i & θ, then the bracketed term should be zero. It is possible if

$$r_1=R\cos i \quad \text{and} \quad r_2=R\cos\theta$$

This concludes that S & S' lie on the circumference of the circle whose diameter is R and this circle is called Rowland's circle.

Eagle's Mounting:



In this mounting the grating G is capable of rotating about a vertical axis and can be moved along a track T. The whole thing is enclosed in a light tight box at the other end of which a plate holder capable of rotation about a vertical axis is mounted. The slit S lies on the side of the box at such a position that its virtual image formed by reflection at the totally reflecting prism at O (which also lies at Rowland circle) just below the centre of the photographic plate P. The light after passing through the Prism falls on the grating.

To form sharply focussed spectra the grating is moved along the track T and also rotated about the vertical axis until the Rowland circle passes through O. The plate holder whose axis passes through O is then so rotated that the surface of the plate also lies on the Rowland circle. The slit S and the totally reflecting prism lie below the central horizontal plane through the grating and the plate lies above it, hence the diffracted light from the grating is not obstructed by the prism. To pass from one spectral region to another the grating G is rotated by proper amount and the above adjustments are repeated.

The advantages of this mounting are that it is cheap and quite compact and hence the ~~various~~ variations in temp. can be easily controlled. This is commonly used in vacuum spectrography for the study of spectra in the ultra violet region below 2000 Å.

Zone Plate

What is Zone Plate ? Give the Construction and theory of a Zone Plate. Show that Zone Plate has multiple focii. Compare the Zone Plate with a Convex lens.

ANS. A Zone Plate is a special diffracting Screen based on Fresnel's theory of half-Period Zones and it is so constructed that alternate Zones obstruct the light.

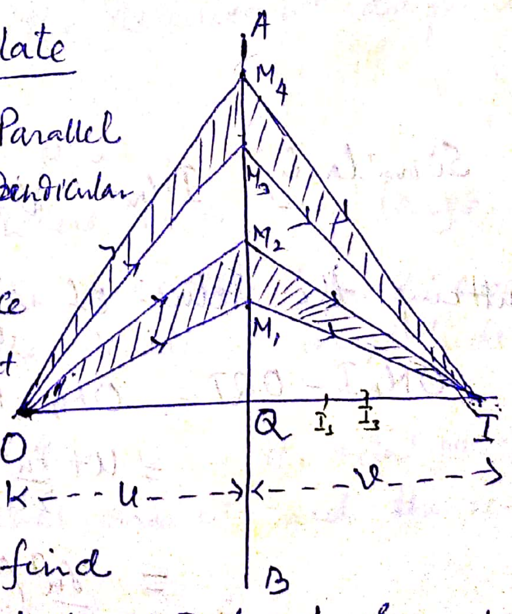
It is Constructed by drawing a series of Concentric Circles on a sheet of Papers with radii Proportional to the square root of the natural numbers like 1, 2, 3, 4 .. etc.

The alternate Zones (either even or odd order Zones) are Covered with black ink and a highly reduced Photograph of the drawing is taken on a plane glass plate

of uniform thickness. The negative thus obtained is a Zone Plate. If the Central half Period Zone is clear (a) it is Called Positive Zone Plate. If the Central Zone is opaque (b) it is Called a negative Zone plate.

Theory of Zone Plate

Let AB be a Plane Parallel transparent Screen perpendicular to the Plane of Paper and O be Point Source of monochromatic light of wavelength λ.



We have to find the resultant amplitude at I due to Secondary wavelets from the various Points of the Screen under the influence of spherical waves diverging from O.

Let us divide the screen into half period zones by choosing point $M_1, M_2, M_3 \cdots M_n$ on the screen such

$$OM_1I - OQI = \lambda/2$$
$$OM_2I - OQI = 2\cdot\lambda/2$$
$$OM_3I - OQI = 3\cdot\lambda/2$$
$$\cdots\cdots\cdots$$
$$OM_nI - OQI = n\lambda/2 \quad \text{——— (1)}$$

(According to Freshel paths of the waves reaching I through two consecutive zones differ by $\lambda/2$)

If concentric circles be drawn on the screen with Q as centre and radii equal to $QM_1 = r_1$, $QM_2 = r_2$ ... $QM_n = r_n$, the area of the first circle is first half period zone, and the area between 2nd circle & first circle is the second half period zone and so on.

Let $OQ = u$ and $QI = v$ then

$$OM_n^2 = QM_n^2 + OQ^2 = u^2 + r_n^2 \quad \text{where } QM_n = r_n.$$

$$\therefore OM_n = (u^2 + r_n^2)^{1/2} = u\left[1 + \frac{r_n^2}{u^2}\right]^{1/2}$$

$$= u + \frac{r_n^2}{2u} \quad \text{---- (2)}$$

(to a first approximation)

Similarly $IM_n = v + \frac{r_n^2}{2v}$ --- (3)

Putting the value of eqn. (2) & (3) in (1), we get

$$OM_nI - OQI = OM_n + M_nI - OQI = n\lambda/2$$

$$= u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} - (u+v)$$

$$= \frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) = n\lambda/2$$

Or $r_n^2 = \dfrac{uvn\lambda}{u+v}$ ——— (4)

$$\therefore \; r_n = \left(\frac{u v \lambda}{u+v}\right)^{\frac{1}{2}} \sqrt{n}$$

ie. $r_n \propto \sqrt{n}$ ie the radii of circles are proportional to square root of natural numbers.

The area of the nth half Period zone is

$$\pi r_n^2 - \pi r_{n-1}^2 = \pi \left\{ \frac{u v n\lambda}{u+v} - \frac{u v (n-1)\lambda}{u+v} \right\}$$

$$= \frac{\pi u v \lambda}{u+v} \quad \text{—(5)}$$

From this eqn (5) it is evident that the area of all the zones are equal. If u becomes infinite we shall have Plane waves falling on the screen. The area of each half Period zone reduces to $\pi v \lambda$

Let $R_1, R_2, R_3 \ldots$ be the amplitudes at I due to Secondary wavelets from 1st, 2nd, 3rd .... zones respectively. Due to obliquity factor $(1+\cos\theta_n)$ the magnitude of $R_1, R_2, R_3 \ldots$ are in decreasing order with the increase of n. Now the amplitudes from alternate zones will have the opposite phases. So, the resultant amplitude at I is given by

$$R = R_1 - R_2 + R_3 - R_4 + \ldots$$

$$= \tfrac{1}{2} R_1 \quad \text{where } n \text{ is very large.}$$

If we intercept the wavelets from the even numbered zones, the Resultant amplitude at I is $R' = R_1 + R_3 + R_5 + \ldots$

$$= \tfrac{1}{2} n R_1 \quad \text{where } n \text{ is the total No. of Zones.}$$

Again if odd zones are blocked, the resultant amplitude at I is

$$R' = R_2 + R_4 + R_6 + \ldots$$

Thus we see that in both cases the resultant amplitude at I is many times greater than that of entire zones.

In other words we say that I will be the image of O. From eq. (4), the relation between u & v is given by.

$$\frac{1}{u}+\frac{1}{v}=\frac{n\lambda}{r_n^2} \quad -(6)$$

If O lies at infinity and v = f, the focal length

Hence $\frac{1}{f}=\frac{n\lambda}{r_n^2}$ or $f=\frac{r_n^2}{n\lambda}$

This result is similar to that for a lens. Thus with regard to light from a luminous point on the axis of a zone plate, it acts like a lens.

Comparison with a Convex lens

Similarities

(I) Both form real image of an object on the side opposite to that of the object.

(II) The relelation between the conjugate distance are similar for both.

(III) Both show chromatic aberration because the focal length varies with λ for both.

Dissimilarities

(I) For a given wavelength of light, a convex lens has a fixed focal length given by $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
But a zone plate for the same type of light has multiple focal lengths and therefore forms a series of images (or focii) of decreasing intensities for a point source because the no. of half period zones contained in an area depend upon the position of the screen (i.e. V), and for each such positions, there is an odd no. of half period elements.

(II) For a convex lens all the rays reaching the image point have the same optical path but for a zone plate the rays reaching the image-point through two successive transparent zones have a path diff. equal to λ.

(III) In a zone plate each colour has its own seperate focus, the red being nearer the plate than the violet

Since $f = r_n^2/n\lambda$ and $\lambda_r > \lambda_v$. Hence $f_r < f_v$.
In case of a ~~lens~~ Convex lens, however it is just the reverse, red being less refrangible than violet comes to focus farther away from the lens i.e. $f_r > f_v$

(IV) The zone plate also acts ~~as~~ a simultaneously as a concave lens unlike the lens it has multiple virtual focii on the side of the source.

(V) The image due to a zone plate is less intense than that due to convex lens.

Multiple focii of a zone plate

A zone plate has a no. of focii which depends upon the number of zones used as well as the wavelength of the light used. For $u = \infty$, $f = v$ and the radius of the nth circle will be given $r_n^2 = Vn\lambda$ (Putting $u = \infty$ in (5)).

And area of nth half period zone $= \pi v \lambda$.
Hence in this case light will be brought to focus at a point I at a distance $V = r_n^2/n\lambda$, for which area of each zone is $\pi v \lambda$. The point given by this relation is known as primary focus or the first order focal ~~length~~ point and is the most intense point.

There is a series of focii unlike a convergent lens, of diminishing intensities as we go along the axis towards the zone plate. The absence of these focii is simply due to the reason that the area of each zone diminishes as points nearer to the plate are concerned.

Let us therefore consider a point I_3 at a distance $v = r_n^2/3n\lambda$ from the zone plate. If the zone plate be imagined to be divided into half period elements, the area of each element will be one third the area of each zone. Hence each zone will contain three half period elements corresponding to I_3. The 2nd black zone intercepts 4th, 5th, and 6th half period elements while the wavelets from 7th, 8th and 9th half period elements contained in the original third half-period zones are transmitted & so on.

Therefore the resultant amplitude at I_3 will be

$$S = (S_1 - S_2 + S_3) + (S_7 - S_8 + S_9) + (S_{13} - S_{14} + S_{15}) + \cdots$$

$$= \left\{ S_1 - \tfrac{1}{2}(S_1 + S_3) + S_3 \right\} + \left\{ S_7 - \tfrac{1}{2}(S_7 + S_9) + S_9 \right\} + \cdots$$

$$= \tfrac{1}{2}(S_1 + S_3 + S_7 + \cdots) \text{ approx.}$$

where $S_1, S_2, S_3 \cdots$ are roughly one third of R_1, $R_2 - R_1$ etc. Hence Point I_3 is of Max^m. intensity which is ~~analogous~~ therefore, the second Point but it is much less intense than I. The light rays transmitted through each successive transparent zone have a Path diff. of 3λ. Hence I_3 is also called the third order focal point.

Similarly it can be shown that Points I_5, I_7 -- etc distant $\frac{r_n^2}{5n\lambda}$, $\frac{r_n^2}{7n\lambda}$ --- from the zone plate are a series of images of the Point object O, but of successively diminishing intensities. Hence we conclude that a zone plate has multiple foci. Unlike a convex lens, the various focal lengths will be $\frac{r_n^2}{n\lambda}$, $\frac{r_n^2}{3n\lambda}$, $\frac{r_n^2}{5n\lambda}$ --- etc.

Production and detection of Plane, Circularly and elliptically Polarised light.

Production of Plane Polarised light

For producing plane Polarised light, a beam of ordinary light is passed through a nicol Prism, it splits up into two Components - E-ray and O-ray. The O Component is totally reflected at Canada balsam layer and is absorbed by the sides of the tube Containing the nicol. The E Component Passes through the Nicol Prism & emerges out. The emergent beam is plane Polarised.

Detection: To detect Plane Polarised light, it is allowed to Pass through another Nicol Prism rotating about the direction of Propagation of light. If the intensity of emergent light varies and vanishes twice in each rotation, then it is plane Polarised.

Production of Circularly Polarised light

It is produced by allowing Plane Polarised light to fall normally on a quater waveplate such that the vibrations in the incident light make an angle of 45° with optic axis of the Plate. In this case the incident light is divided into E and O Components of equal amplitude $A\cos 45^\circ$ and $A\sin 45^\circ$.

Let $A\cos 45^\circ = A\sin 45^\circ = a$. As the quarter wave plate introduces a Phase difference of $\pi/2$, the two components may be written as

$$x = a\sin(\omega t + \pi/2) = a\cos\omega t \;\; \& \;\; y = a\sin\omega t$$

The resultant vibration is therefore given by
$x^2+y^2=a^2$ which represents a circle.
Hence the light emerging from $\lambda/4$ plate is circularly polarised.

Detection : The circularly polarised light when seen through a rotating nicol prism shows no variation. In this respect it resembles unpolarised light. Hence to detect circularly polarised light it is first pass normally through a $\lambda/4$ plate (which converts it into a plane polarised light) and then through a nicol. If the light vanishes twice in each rotation of nicol prism then it is circularly polarised light otherewise it is unpolarised.

Production of elliptically polarised light

The elliptically polarised light is produced by sending plane polarised light normally through quarter wave plate and that the direction of vibration in the incident light makes an angle θ with the optic axis of the plate.
Let A be the amplitude of vibrations of the incident light. Inside the plate, the light is divided into two plane polarised waves, one wave parallel to the optic axis (E wave) and other $\perp r$ to the optic axis (O-wave). The amplitudes of E and O waves will be $A\cos\theta$ and $A\sin\theta$ respectively. The phase difference at the point of entering is zero and at the point of emergence is $\pi/2$.

let $A\cos\theta = a$ and $A\sin\theta = b$. If the axes of x & y are taken along & $\perp r$ to the optic axis, then the equations of E and O waves may be written as

$$x = a\sin(\omega t+\pi/2) = a\cos\omega t \quad \& \quad y = b\sin\omega t$$

Eliminating t in the above two equations, we get the equation of resultant vibration as

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

This equation represents an elliptic vibration, hence the emergent light is in general elliptically Polarised light. However when $\theta=0$, $b=0$ and the emergent light is Plane Polarised with the vibration Parallel to the optic axis. When $\theta=90^\circ$, $a=0$ and the emergent light is Plane Polarised with vibrations ⊥r to the optic axis. When $\theta=45^\circ$, $a=b$. and the resultant vibration is Circular so that the emergent light is Circularly Polarised.

Hence in order to produce elliptically Polarised light θ must be different from 0°, 45° & 90°.

Detection: If the light is incident on a Nicol Prism and the intensity varies from a Maximum to a minimum then the light is either elliptically Polarised or a mixture of Polarised and unpolarised light.

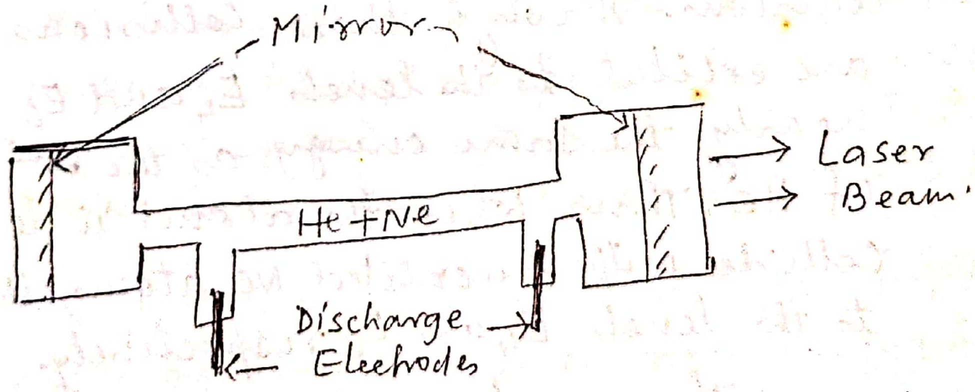
Now, the light is made to fall on a quarter wave plate and then on a rotating Nicol Prism. If the light vanishes twice each rotation of the Nicol Prism, it is Polarised elliptically otherwise it is a mixture of Polarised light + unpolarised light.

He - Ne Laser

Laser is a device which amplifies or generates radiation by means of the stimulated emission process. Its ~~mo~~ name is derived from the initials of Light Amplification by Stimulated Emission of Radiation. All lasers require an active medium for amplification in a narrow frequency region by population inversion achieved between a pair of energy levels. Under the condition of population inversion, the amplification of a wave occur as it passes through the active medium. This amplification is coherent.

The He-Ne laser is the most widely used laser with continuous power output in the range of a fraction of mW to tens of mW. It is easy to construct and is reliable in operation. It was the first gas laser to be operated successfully. It was fabricated by Ali Javan in 1961. It is four-level laser in which population inversion is achieved by electric discharge.

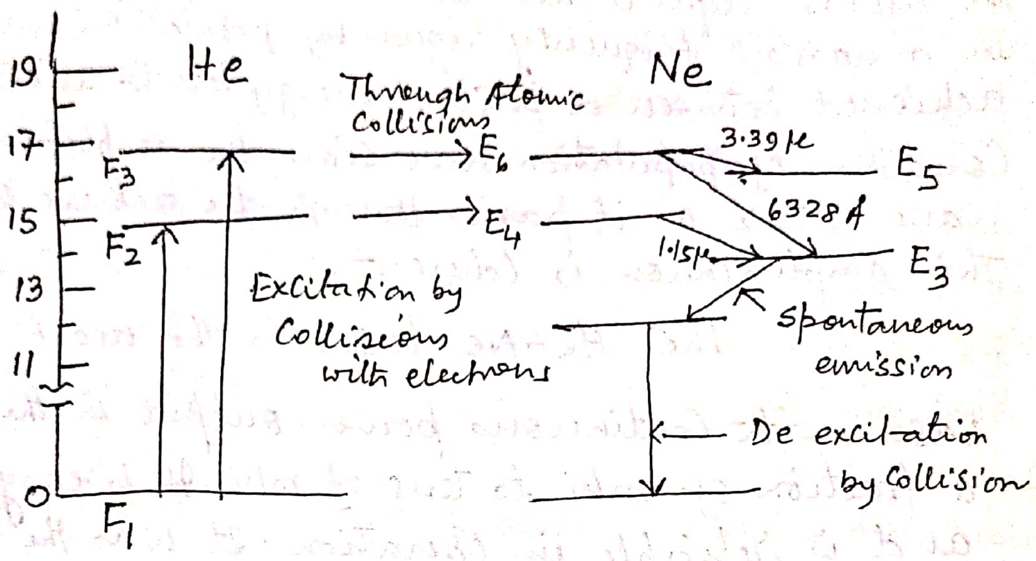
The He-Ne laser consists of a mixture of He and Ne in the ratio of about 10:1 placed



inside a long narrow tube (See fig). The pressure inside the tube is 1 Torr (= 1mm of Hg). The gas system is enclosed between a pair of plane mirrors so that a resonator system is formed.

one of the mirror is of very high reflectivity while the other is partially transparent so that the energy may be Coupled out of the system.

The Lasing action of He-Ne laser has been shown by the energy level diagram. The first few energy levels have been taken for convenience of fig.



When an electric discharge is passed through the gas, the elections travelling down the tube Collide with He atoms and excite them (ground state F_1) to the levels marked F_2 and F_3. These levels are metastable i.e. He atoms excited to these states where stay there levels for a Sufficiently long time before losing energy through collisions. Through these collisions, the Ne atoms are excited to the levels E_4 and E_6 which have nearly the same energy as the ~~He~~ levels F_2 and F_3 of He. Thus when the atoms in levels F_2 and F_3 Collide with unexcited Ne atoms, they raise them to the levels E_4 and E_6 respectively.

Thus, we have following two step processes—

1. He atom in ground state F_1 + Collision with electrons

$\longrightarrow$ He atom is excited to F_2 or F_3 state + electron with Lesser kinetic energy

2. He atom in excited State F_3 + Ne atom in the ground state $\longrightarrow$ He atom in ground state + Ne atom in excited state E_6

Similarly

He atom in excited state F_2 + Ne atom in the ground state

$\longrightarrow$ He atom in ground state + Ne atom is excited to E_4 state.

This results in the sizeable population of the levels E_4 and E_6. The population in these levels happen to be much more than those in the lower levels E_3 and E_5. Thus a state of population inversion is achieved and any spontaneously emitted photon can trigger laser action in any of the three transitions as shown in fig. The Ne atoms then drop down from the lower laser level E_2 through spontaneous emission. From the level E_2 the Ne atoms are brought back to the ground state through collisions with the walls.

The transition from E_6 to E_5, E_4 to E_3 and E_6 to E_3 result in the emission of radiation having wavelength 3.39 μm, 1.15 μm and 6328 A° respectively. The 6328 A° transition correspond to the well known red light of He-Ne laser. Rest two are not in visible region.

A proper selection of different frequencies may be made by chosing end mirrors having high reflectivity over only the required wavelength range. The pressures of two gases must be so chosen that the condition of population inversion is not quenched. Thus, the condition must be such that there is an efficient transfer of energy from He to Ne atoms. Referring to the fig, it may be mentioned that actually there are a large ~~of~~ no. of levels grouped around E_2, E_3, E_4, E_5 & E_6. only those levels are shown in the fig which corresponds to the important laser action.

Gas lasers are, in general, found to emit light, which is more directional and more monochromatic. This is because of the absence of such effects as crystalline imperfection, thermal distortion and scattering, which are present in solid state laser. Gas lasers are capable of operating continuously without need for cooling.

Applications: Lasers have tremendous uses. Modulated laser beams have been used for communication. Lasers have been used by the medical professionals in surgery, where retinal tissues is cauterised to weld detached retina. They have been used by surveyors and engineers for critical allignment as well as for ranging in metrology and determining the distance of the moon. Attenuation and scattering of lasers beams have been used in atmospheric research. High power lasers have been used to cut through diamond and steel plates and to initiate thermo nuclear reactions. It has been used in the production and reasearch with holograms. Also used to study Biological samples. During war-time lasers are used to detect and destroy enemy missiles. Now laser rifles, laser bombs are used.
There are further many more applications.