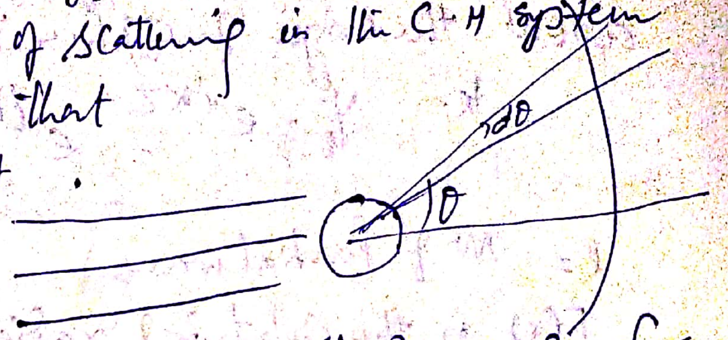
n-p scattering

n-p scattering Expt in the energy range upto 10 Mev Reveals that angular distribution of scattering in the C.M system is isotropic. It implies that scattering is independent of l values i.e. only $l=0$ value is involved.



This suggest that nuclear forces are short range force.

Partial wave analysis of n-p scattering

In the C.M system the wave eqn for n-p scattering is given by

$$\nabla^2\psi + \frac{M}{\hbar^2}\left[E - V(r)\right]\psi = 0 \quad \text{—(1)}$$

where $V(r)$ is the scattering potential. At large distance from scattering centre (i.e. $P.A \to 0$), the expected form of solution of eqn. (1) is

$$\psi = e^{ikz} + \frac{e^{ikr}}{r} f(\theta) \quad \text{—(2)}$$

where the first term represent the plane wave travelling along z-axis towards scattering centre and the 2nd term represent an outgoing scattered spherical wave. $f(\theta)$ is the amplitude of scattered wave in the direction θ w.r.t. the incident beam. The wave no.

$$K = \sqrt{\frac{ME}{\hbar^2}} = \frac{Mv}{\hbar} \quad \text{—(3)}$$

. It is obvious that

$$\psi = \psi_{inc} + \psi_{sc} \quad \text{—(4)}$$

let us first take the wave eqn in absence of scattering potential

$$(\nabla^2 + k^2)\psi = 0 \quad \text{—(5)}$$

The solution has the for

$$\psi = e^{ikz} \quad \text{—(6)}$$

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According to Lord Rayleigh a Plane wave can be represented by the sum of a series of spherical waves about the origin of spherical co-ordinate

$$\psi_i = e^{ikz} = e^{ikr\cos\theta} = \sum_{l=0}^{\infty} F_l(r) P_l(\cos\theta) \quad \text{—} (7)$$

l = No of Partial waves

$F(r)$ is the solution of radial part, $P_l(\cos\theta)$ legendre Polynomial

for $l=0$, the solution becomes

$$\psi_0 = F_0(r) P_0(\cos\theta) = \frac{\sin kr}{kr} = \frac{e^{ikr} - e^{-ikr}}{2ikr} \quad \text{—} (8)$$

Now P.E. $V(r)$ is taken into account. However it does not affect the incoming part of the wave at large distance. The scattering potential only change the phase of outgoing wave. Now the wavefn (8) becomes

$$\psi_0 = \frac{e^{i(kr+2\delta_0)} - e^{-ikr}}{2ikr}$$

$$= e^{i\delta_0} \frac{\sin(kr+\delta_0)}{kr} \quad \text{—} (9)$$

Now the total wave fn. in the presence of scattering potential will be the sum of ψ_{inc} & difference of eqn: (9) & (8), Hence the complete wave fn. is

$$\psi = e^{ikz} + \frac{e^{i(kr+2\delta_0)} - e^{ikr}}{2ikr}$$

$$= e^{ikz} + \frac{e^{i(kr+\delta_0)}}{kr} \sin\delta_0 \quad \text{—} (10)$$

Comparing eqn (10) & (2), we get

$$\underset{l=0}{f(\theta)} = \frac{e^{i\delta_0}\sin\delta_0}{K} \quad \text{--- (11)}$$

The scattering cross section in terms of Phase shift will be given by

$$\sigma_{sc} = 4\pi\,|f(\theta)|^2$$

$$= 4\pi \sin^2\delta_0 / K^2 \quad \text{--- (12)}$$

It is seen that s-wave scattering is isotropic because they do not contain angular dependence term.
In general if Phase shift is δ_l, total scattering cross section will be given by

$$\sigma_l = \frac{4\pi}{K^2}\sum_{l=0}(2l+1)\sin^2\delta_l \quad \text{--- (13)}$$

Also the value of Phase shift can be determined which comes to be

$$\sin^2\delta_0 = \frac{1}{1+\frac{B}{E}}$$ & the total scattering

Cross section $\sigma_{sc(0)} = \frac{4\pi}{K^2}\frac{E}{E+B} = \frac{4\pi\hbar^2}{M(E+B)}$ — (14)

where B is the B.E. The eqn (14) is in satisfactory agreement with expt. Result provided $E \gg B$.

Scattering length

For thermal neutrons, of energy $E \to 0$, the scattering amplitude $f(\theta)$ [for $l=0$]

takes the approximate form given by

$$f(0) = \frac{e^{i\delta_0}\sin\delta_0}{k}$$

when the incident neutrons have got very small energy i.e. when $k \to 0$, $e^{i\delta_0} \to 1$ as δ_0 will be very small

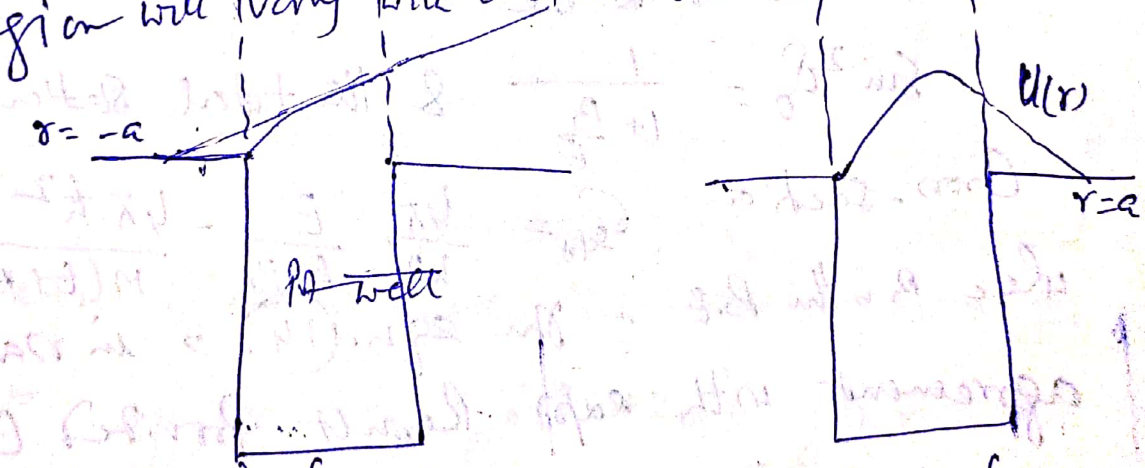
$$\therefore f(\theta) = \frac{\delta_0}{k} = -a \quad \text{—— (15)}$$

Here a is called the scattering length and in limiting case gives a real value. There will be two kinds of situations at the surface of Pot. well where $V(r) = 0$

$$\psi_{total} = \frac{e^{i\delta_0}}{kr}\sin(kr+\delta_0) = \frac{1}{kr}\times(kr+\delta_0)$$

$$\text{or } r\psi_{total} = U(r) = r + \frac{\delta_0}{k} = r - a \quad \text{—— (16)}$$

The amplitude of the wave fn $U(r)$ in the outer region will vary with distance r as a st line



Scattering from a unbound state gives a -ve scattering length

Scattering from a bound sp state gives a +ve scattering length

The -ve scattering length is for unbound system and +ve scattering length is for bound system.
Further $a = -\frac{\delta_0}{k}$ implies that +ve phase shift takes place for -ve scattering length and -ve phase shift for +ve scattering length.

Now the scattering cross section for zero energy neutrons will have value

$$\sigma_0 = 4\pi a_0^2$$