F. Ahmad

The Poision Distribution

As we know from bionomial distribution that the possibility of events occured and not occured is only important. For example in hokey match how many goal scored is important. Death due to brain haemorrhage in a town may only be recorded.

Thus poision distribution is limiting case for Binomial distribution when p is a very a small number and n is so big that np is finite constant, means equal to m. According to this distribution probability of r success is as follows:

$$p(x=r) = \frac{m^r e^{-m}}{r!}$$

Here m is parameter of distribution
$r = 0, 1, 2, 3, \ldots$ accepts the values
$!$ = Factorial

In bionomical distribution we use as:

$$f(r) = {}^nC_r\, p^r q^{n-r}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \cdot p^r q^{n-r}$$

$$= n^n\left(1-\frac{1}{n}\right)^n\left(1-\frac{2}{n}\right)\cdots n\left(1-\frac{r-1}{n}\right) \times p^r (1-p)^{n-r}$$

because $p+q=1$ $\therefore q = 1-p$

$$= \frac{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right)}{r!} \times n^r p^r (1-p)^{n-r}$$

$$= \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{r-1}{n}\right)$$

$$\times (np)^r \left(1-\frac{np}{n}\right)^{n-r}$$

Now taking limit when $n \to \infty$ and $p \to 0$ by which $np = m$, we

$$f_r = \frac{1}{r!} \times m^r \times \underset{n\to\infty}{Lt} \left(1-\frac{m}{n}\right)^{n-r}$$

But $\underset{n\to\infty}{Lt} \left(1-\frac{m}{n}\right)^{n-r}$

$$= Lt \left(1-\frac{m}{n}\right)^n \times \underset{n\to\infty}{Lt} \left(1-\frac{m}{n}\right)^{-r}$$

$$= e^{-m} \times 1 = e^{-m}.$$

$\therefore$ From (1)

$$f(r) = \frac{1}{r!} m^r e^{-m}$$

i.e. Probability of r success when $n \to \infty$ and $p \to 0$

$= \frac{1}{r!} m^r e^{-m}$

It means $P(X=r) = \frac{1}{r!} m^r e^{-m}$

Thus, $r = 0$ Keeping 1, 2, 3, 4

Values of 0, 1, 2, 3 probabilities

$e^{-m}, \frac{me^{-m}}{1!}, \frac{m^2 e^{-m}}{2!}, \frac{m^3 e^{-m}}{3!} \ldots$

In Tabular form this can be kept as:

x, How many times

0

1

2

3

....

r

......