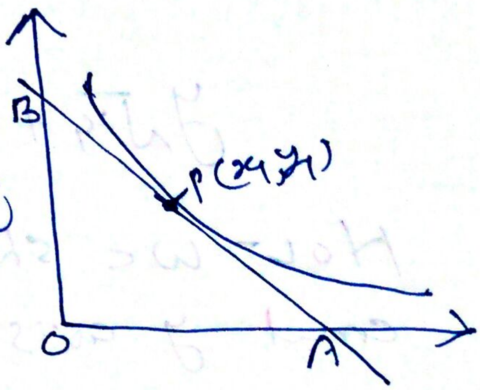
Exp. Show that the sum of the intercepts of the tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the Co-ordinate axes is constant.

Sol. Let $P(x_1, y_1)$ be any point on the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$

∵ Then the Equation of the tangent at $P(x_1, y_1)$ will be




$$y-y_1=\left(\frac{dy}{dx}\right)_{\substack{x=x_1\\y=y_1}}(x-x_1) \quad -① $$

given curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{x}}+\frac{1}{2}\frac{1}{\sqrt{y}}\cdot\frac{dy}{dx}=0$$

$$\frac{1}{\sqrt{y}}\frac{dy}{dx}=-\frac{1}{\sqrt{x}}$$

$$\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{\substack{x=x_1\\y=y_1}}=-\frac{\sqrt{y_1}}{\sqrt{x_1}}$$

Putting in ① we get

$$y-y_1=-\frac{\sqrt{y_1}}{\sqrt{x_1}}(x-x_1)$$

$$y\cdot\sqrt{x_1}-y_1\cdot\sqrt{x_1}=-\sqrt{y_1}\cdot x+\sqrt{y_1}\cdot x_1$$

$$y\cdot\sqrt{x_1}+x\cdot\sqrt{y_1}=y_1\cdot\sqrt{x_1}+\sqrt{y_1}\cdot x_1$$

$$y \cdot \sqrt{x_1} + x \cdot \sqrt{y_1} = \sqrt{x_1} \cdot \sqrt{y_1} \left( \sqrt{y_1} + \sqrt{x_1} \right) \quad \text{--- (11)}$$

$\because$ $(x_1, y_1)$ lies on the curve $\sqrt{x} + \sqrt{y} = \sqrt{9}$

then $\sqrt{x_1} + \sqrt{y_1} = \sqrt{9}$

Putting in (11)

$$y \sqrt{x_1} + x \cdot \sqrt{y_1} = \sqrt{x_1} \cdot \sqrt{y_1} \left( \sqrt{9} \right) \quad \text{--- (3)}$$

Now we shall find out the intercept on x axis and y axis of the st. line (3)

Let the st. line (3) cuts the x axis at A and the y axis at B

i.e The y-co-ordinate of A = 0 i.e y = 0 put in (3)

$$x \cdot \sqrt{y_1} = \sqrt{x_1} \sqrt{y_1} \sqrt{9}$$

$$x = \sqrt{x_1} \cdot \sqrt{9}$$

i.e the intercept OA on the x axis $= \sqrt{x_1} \sqrt{9}$

Similiarly to find out the intercept on y axis put $x = 0$ in (3) then

$$y = \sqrt{y_1} \sqrt{9}$$

i.e the intercept of OB on the y axis $= \sqrt{y_1} \sqrt{9}$

Sum of intercepts OA + OB is

$$\sqrt{x_1} \cdot \sqrt{9} + \sqrt{y_1} \cdot \sqrt{9}$$

$$= \sqrt{9} \left( \sqrt{x_1} + \sqrt{y_1} \right) = \sqrt{9} \cdot \sqrt{9} = 9$$

= constant. Hence the result