System of Linear Equctions -

A system of linear equations is a list of linear equations with the same unknowms $(x_1, x_2, x_3 \ldots x_n)$. A system of m linear equations $(L_1, L_2, \ldots L_m)$ in n unknowns $x_1, x_2, \ldots x_n$ can be put in standard form:- $AX = B$

$$\left.\begin{aligned} L_1 &\rightarrow a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots a_{1n}x_n = b_1 \\ L_2 &\rightarrow a_{21}x_1 + a_{22}x_2 + \cdots\cdots a_{2n}x_n = b_2 \\ &\cdots\cdots \\ L_m &\rightarrow a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots a_{mn}x_n = b_m \end{aligned}\right\} \text{—} (1)$$

where a_{ij} and b_i are constants for $i = 1, 3 \ldots m$, $j = 1, 3 \ldots n$

The system of L. Eqn (1) is called $m \times n$ (m by n) system

⇒ The system (1) is said to be homogeneous if all constant terms are zero, that is if $b_1 = 0, b_2 = 0, \ldots b_m = 0$. otherewise the system is said to be non-homo

⇒ A solution of the system (1) is a list of values for the unknowns (n).

⇒ The system (1) of linear Eqn is said to be consistent if it has one or more solutions and it is said to be inconsistant if it has no solution.

* Coefficient Matrix of a System (1)

$$A = \begin{bmatrix} a_{11} & a_{12} & - & a_{1n} \\ a_{21} & a_{22} & - & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}$$

rank of Coeff. Matrix $\rho(A)$ and rank of augmented matrix $\rho(M)$ is equal

i.e $\rho(A:B) = \rho(A)$

or $\rho(M) = \rho(A)$

* Augmented matrix of the system (1)

$$M = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & : & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & : & b_2 \\ \cdots & \cdots & & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & : & b_m \end{bmatrix}$$

i.e $M = [A : B]$

System of Linear Homogeneous Equations.

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots\ a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots\ a_{2n}x_n = 0 \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots\ a_{mn}x_n = 0 \end{aligned}\right\} \text{--- } ②$$

Coefficient matrix A

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix} \qquad M = \left[\begin{array}{ccc|c} a_{11} & a_{12} \cdots & a_{1n} & 0 \\ a_{21} & a_{22} \cdots & a_{2n} & 0 \\ \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \\ a_{m1} & a_{m2} \cdots & a_{mn} & 0 \end{array}\right]$$

(i) $\rho(A:B) = \rho(A) = n$ then trivial solution i.e. $x_1 = x_2 = x_3 \cdots = 0$

$\Rightarrow$ To find the rank of A by reducing it to the triangular form or Echelon form by Elementary row orperations. (i) If the rank $A = n$, then have only a trivial solution $x_1 = x_2 = \cdots = 0$

$\rho(A) < n \Rightarrow$ (ii) If the rank of $A < n$, then have infinite non-trivial solutions.

Elementary Row operations / Transformation

① $R_i \leftrightarrow R_j$ Interchange

(ii) $R_i \rightarrow kR_i$ Replace $k \neq 0$

(iii) $R_j \rightarrow R_j \pm kR_i$ Replace by another Row with sum/subtract of a multiple of itself or another row.

<u>Rank of a Matrix $\rho(A)$</u>. $\rightarrow$ A matrix is said to be of rank r when

(i) it has at least non-zero minor of order r

(ii) and every minor of order higher than r vanishes i.e $|A| = 0$

(*) The maximum number of row of a matrix A is linearly independent (i.e $|A| \neq 0$) is called the rank of A i.e $\rho(A)$.