

Exp. Solve the given differential Equation.

$$(x-y) dx + (x-y+2) dy = 0$$

Solution Given differential Eqn.

$$(x-y) dx + (x-y+2) dy = 0$$

$$\frac{dy}{dx} = \frac{y-x}{x-y+2} \quad \text{or} \quad \frac{dy}{dx} = \frac{-x+y+0}{x-y+2} \quad \text{as} \quad \frac{a_1x+b_1}{a_2x+b_2}$$

$$\Rightarrow a_1 = -1 \quad b_1 = 1 \quad \Rightarrow \frac{a_1}{b_1} = -1$$

$$a_2 = 1 \quad b_2 = -1 \quad \Rightarrow \frac{a_2}{b_2} = -1$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} \quad \text{Thus case II is applicable}$$

$$\text{or} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \quad \text{s.c.} \quad a_1 b_2 - a_2 b_1 = 0 \quad \text{Then}$$

the method fails as h and k become infinite or indeterminate.

Thus. put $y-x = t$

$$\frac{dy}{dx} - 1 = \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dt}{dx} + 1$$

$$\frac{dt}{dx} + 1 = \frac{t+0}{-t+2} \quad \Rightarrow \quad \frac{dt}{dx} = \frac{t}{2-t} - 1 = \frac{t-2+t}{2-t}$$

$$\frac{dt}{dx} = \frac{2t-2}{2-t} = \frac{2(t-1)}{2-t}$$

$$\frac{2-t}{2(t-1)} dt = dx$$

$$\frac{2}{2(t-1)} dt - \frac{t}{2(t-1)} dt = dx$$

$$\frac{dt}{t-1} - \frac{(t+1-1)}{2(t-1)} dt = dx$$

$$\frac{dt}{t-1} - \frac{(t-1)}{2(t-1)} dt - \frac{dt}{2(t-1)} = dx$$

$$\log(t-1) - \frac{1}{2}t - \frac{1}{2}\log(t-1) = x + c$$

$$\frac{1}{2}\log(t-1) - \frac{1}{2}t = x + c$$

$$\frac{1}{2}\log(y-x-1) - \frac{1}{2}(y-x) = x + c$$

$$\frac{1}{2}\log(y-x-1) = \frac{1}{2}(y-x) + x + c$$

$$\frac{1}{2}\log(y-x-1) = \frac{1}{2}(x+y) + c$$

$$x+y+c' = \log(y-x-1)$$

$$x+y - \log(x-y+1) = c'$$

log m.n
 $\therefore \log(x) \cdot (x-y+1)$
 $\Rightarrow \log(x) + \log(x-y+1)$
 $\log 1 = 0$

Exercise - Solve the differential Equ.

$$(x+2y+1)dx + (2x+4y+3)dy = 0$$

Hint - $\frac{a_1}{b_1} = \frac{a_2}{b_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Thus put $x+2y = t$
 $1+2\frac{dy}{dx} = \frac{dt}{dx}$ in above Eqn then solve.