Second Form of Tangent :- Let the Equation of the curve by given by $f(x,y)=0$

We know from the partial differentiation that

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

Thefore putting the value of $\frac{dy}{dx}$ in the Eqn.

$$Y-y = \frac{dy}{dx}(X-x)$$

$$Y-y = \frac{-\partial f/\partial x}{\partial f/\partial y}(X-x)$$

$$(Y-y)\frac{\partial f}{\partial y} = -\frac{\partial f}{\partial x}(X-x)$$

$$(X-x)\frac{\partial f}{\partial x} + (Y-y)\frac{\partial f}{\partial y} = 0$$

This is the second form of the Eqution of the tangent.

Exp. Find the equation of tangent at (a,b) to be curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

Sol. Given curve

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n \quad \text{--- (1)}$$ the find

$\frac{dy}{dx}$ from this curve.

Differentrating, we get

$$n\left(\frac{x}{a}\right)^{n-1}\cdot\frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1}\cdot\frac{dy}{dx}\cdot\frac{1}{b} = 0$$

$$\frac{n}{b}\left(\frac{y}{b}\right)^{n-1}\frac{dy}{dx} = -\frac{n}{a}\left(\frac{x}{a}\right)^{n-1}$$

$$\frac{dy}{dx} = -\frac{b}{a}\left(\frac{x}{a}\right)^{n-1}\cdot\left(\frac{b}{y}\right)^{n-1}$$

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1}\cdot\frac{b^n}{a^n}$$

$$\left(\frac{dy}{dx}\right)_{\substack{x=a\\y=b}} = -\left(\frac{a}{b}\right)^{n-1}\cdot\left(\frac{b}{a}\right)^{n}$$

$$\left(\frac{dy}{dx}\right)_{\substack{x=a\\y=b}} = -\frac{b}{a}$$

∴ The Equation of tangent at (a, b)

$$y-b = \left(\frac{dy}{dx}\right)_{\substack{x=a\\y=b}}(x-a)$$

$$y-b = -\frac{b}{a}(x-a)$$

$$ay-ab = -bx+ab$$

$$bx+ay = 2ab$$

$$\frac{x}{a}+\frac{y}{b} = 2$$