Matrices

Properties of matrix operations:-

2 (i) show that matrix addition is commutative

Let $A = (a_{ij})_{m,n}$ and $B = (b_{ij})_{m,n}$ be matrices of the same type. Then $A+B$ and $B+A$ are both defined.

We have to show that $A+B = B+A$

Now, $A+B \overset{\text{def.}}{=} (a_{ij}+b_{ij})_{m,n}$ [$\because$ addition of scalars is commutative]

$= (b_{ij}+a_{ij})_{m,n}$

$\overset{\text{def.}}{=} B+A$

Hence matrix addition is commutative.

(ii) show that matrix addition is associative

Let $A = (a_{ij})_{m,n}$, $B = (b_{ij})_{m,n}$ and $C = (c_{ij})_{m,n}$ be matrices of same type. Then

$A+B$, $(A+B)+C$, $B+C$ and $A+(B+C)$ are all defined. we have to show that

$$(A+B)+C = A+(B+C)$$

Now, $A+B \overset{\text{def.}}{=} (a_{ij}+b_{ij})_{m,n}$

and $(A+B)+C \overset{\text{def.}}{=} [(a_{ij}+b_{ij})+c_{ij}]_{m,n}$

$= [a_{ij}+(b_{ij}+c_{ij})]_{m,n}$

[$\because$ addition of scalars is associative]

$$= (a_{ij})_{m,n} + (b_{ij} + c_{ij})_{m,n}$$

$$\overset{\text{def.}}{=} A + (B + C)$$

Hence the result.

(iii) Show that matrix multiplication is not Commutative.

Proof :- In order that the matrix multiplication be Commutative. both the following Conditions should be satisfied for any two matrices A and B :-

(a) If the product AB is defined then the product BA is also defined; and

(b) $AB = BA$

We note that if A is an $m \times n$ matrix and B is an $n \times p$ matrix $(p \neq m)$; then AB is defined but BA is not defined.

$\therefore$ In Condition (a) is not satisfied for all matrices A and B.

Moreover, as seen in the following example, even if AB and BA are both defined, the condition (b) may not be satisfied :-

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

Here A is a 2×2 matrix and B is a 2×2 matrix.

$\therefore$ Both AB and BA are defined.

Now $AB = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+1 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

while $BA = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0 & 1+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

and so $AB \neq BA$

Thus even if AB and BA are both defined, they are not equal. Hence matrix multiplication is not, in general commutative.

[observe : In the above example we have seen that $AB = B \not\Rightarrow A$ is a unit matrix; similarly, $BA = A \not\Rightarrow B$ is a unit matrix]