* Predictor-corrector methods are the ones which require function values at $x_n, x_{n-1}, x_{n-2}, \ldots$ for the computation of the function value at x_{n+1}. A predictor formula is used to predict the value of y at x_{n+1} and then a corrector formula is used to improve the value of y_{n+1}. Here, we drive Predictor-corrector formulae which use Newton's backward differences and Milne's method which uses forward differences.

* Adams-Moulton Method-

Newton's backward differences interpolation formula can be written as

$$f(x,y) = f_0 + \frac{n}{\lfloor 1}\nabla f_0 + \frac{n(n+1)}{\lfloor 2}\nabla^2 f_0 + \cdots \quad \text{——— (1)}$$

where $n = \frac{x-x_0}{h}$ and $f_0 = f(x_0, y_0)$

If this formula is substituted in

$$y_1 = y_0 + \int_{x_0}^{x_1} f(x,y)\,dx \quad \text{——— (2)}$$

We get

$$y_1 = y_0 + \int_{x_0}^{x_1}\left[f_0 + \frac{n}{\lfloor 1}\nabla f_0 + \frac{n(n+1)}{\lfloor 2}\nabla^2 f_0 + \cdots\right]dx$$

$$y_1 = y_0 + h\int_0^1\left[f_0 + \frac{n}{\lfloor 1}\nabla f_0 + \frac{n(n+1)}{\lfloor 2}\nabla^2 f_0 + \cdots\right]dn$$

$$y_1 = y_0 + h\left[1 + \frac{1}{2}\nabla + \frac{5}{12}\nabla^2 + \frac{3}{8}\nabla^3 + \frac{251}{720}\nabla^4 \cdots\right]f_0$$

Hence this formula can be used to compute y then

$$y_1^P = y_0 + h\left(1 + \frac{1}{2}\nabla + \frac{5}{12}\nabla^2 + \frac{3}{8}\nabla^3 + \frac{251}{720}\nabla^4 \cdots\right)f_0^P \quad (3)$$

This is called Adames-Bashforth Formula as a predictor formula.

A corrector formula can be derived in a similar manner by using Newton's backward difference formula at f_1.
putting f_1 at f_0 in Eqn (1) or Replacing as f_1

$$f(x, y) = f_1 + n\nabla f_1 + \frac{n(n+1)}{\lfloor 2}\nabla^2 f_1 + \frac{n(n+1)(n+2)}{\lfloor 3}\nabla^3 f_1$$

$+ \cdots \quad (4)$

Substituting Eqn (2) & (4) we obtain

$$y_1 = y_0 + \int_{x_0}^{x_1} \left[f_1 + n\nabla f_1 + \frac{n(n+1)}{\lfloor 2}\nabla^2 f_1 + \cdots\right] dx$$

$$= y_0 + h\int_{-1}^{0} \left[f_1 + n\nabla f_1 + \frac{n(n+1)}{\lfloor 2}\nabla^2 f_1 + \cdots\right] dn$$

$$y_1 = y_0 + h\left[1 - \frac{1}{2}\nabla - \frac{1}{12}\nabla^2 - \frac{1}{24}\nabla^3 - \frac{19}{720}\nabla^4 - \cdots\right] f_1$$

Hence, this formula can be used to correct value, we rewrite it

$$y_1^{(c)} = y_0 + h\left[1 - \frac{1}{2}\nabla - \frac{1}{12}\nabla^2 - \frac{1}{24}\nabla^3 - \frac{19}{720}\nabla^4 - \cdots\right] f_1^{(p)}$$

(5)

This is called Adams Moulton Corrector Method for y_1 and superscript p in f_1^p (right side) indicating that the predicted value of y_1 should be used for computing the value of $f(x_1, y_1)$.

However, it will be convenient to use Eqn (3) and (5) by ignoring the higher-order differences and expressing the lower-order differences in terms of function values. Then Eqn's of (3) and (5) formulae can be written as—

$$y_1^p = y_0 + \frac{h}{24}[55f_0 - 59f_{-1} + 37f_{-2} - 9f_{-3}) \quad (6)$$

$$y_1^c = y_0 + \frac{h}{24}[9f_1^p + 19f_0 - 5f_{-1} + f_{-2}) \quad (7)$$

The general forms of formulae (6) and (7) as

$$y_{n+1}^p = y_n + \frac{h}{24}[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}]$$

$$y_{n+1}^c = y_n + \frac{h}{24}[9f_{n+1}^p + 19f_n - 5f_{n-1} + f_{n-2}]$$

These formulae, expressed in ordinate form are often explicit predictor corrector formulae.

In which the errors are approximately

$\frac{251}{720} h^5 f_0^{(4)}$ & $\frac{-19}{720} h^5 f_0^{(4)}$ for

Eq (6) and (7) respectively.