Langat Singh College, Muzaffarpur
NAAC Grade 'A'
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Plasma Physics – Lecture - 03

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Plasma Shielding

- **Elementary Derivation of the Boltzmann Distribution**
- **Basic principle of Statistical Mechanics:**
- Thermal Equilibrium $\leftrightarrow$ Most Probable State i.e. State with large number of possible arrangements of microstates.

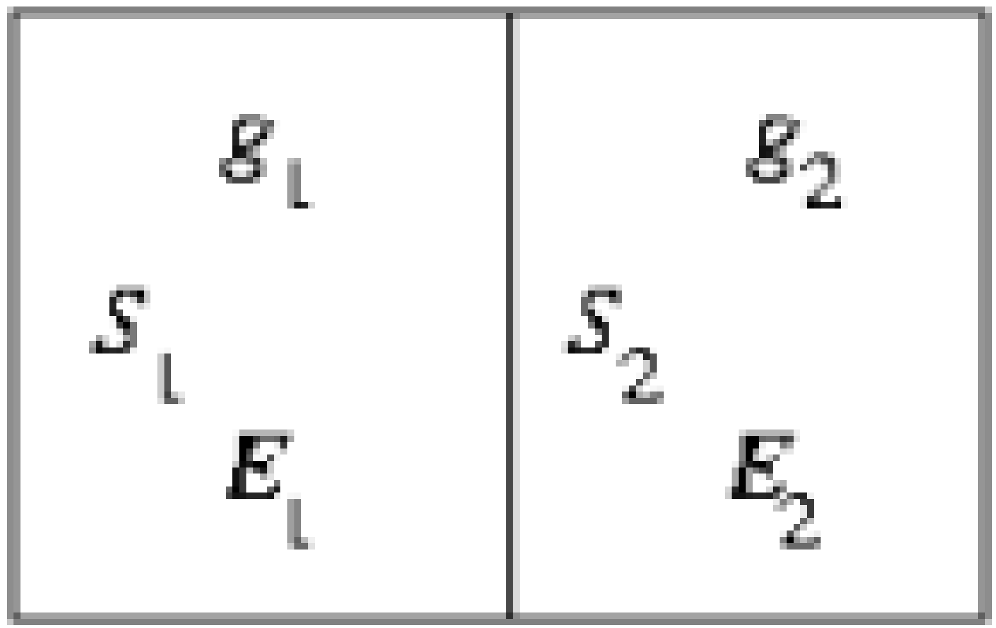

Figure 4: Statistical Systems in Thermal Contact

- Consider two weakly coupled systems S_1, S_2 with energies E_1, E_2.
- Let g_1, g_2 be the number of microscopic states which give rise to these energies, for each system.
- Then the total number of microstates of the combined system is (assuming states are independent)

$$g = g_1 g_2 \tag{10}$$

- If the total energy of combined system is fixed $E_1 + E_2 = E_t$ then this can be written as a function of E_1:

$$g = g_1(E_1)g_2(E_t - E_1) \tag{11}$$

$$\text{and} \quad \frac{dg}{dE_1} = \frac{dg_1}{dE}g_2 - g_1\frac{dg_2}{dE} \tag{12}$$

The most probable state is that for which dg dE_1 = 0 i.e

$$\frac{1}{g_1}\frac{dg_1}{dE} = \frac{1}{g_2}\frac{dg_2}{dE} \text{ or } \frac{d}{dE}\ln g_1 = \frac{d}{dE}\ln g_2 \tag{13}$$

Thus, in equilibrium, states in thermal contact have equal values of g. One defines $\sigma \equiv \ln(g)$ as the Entropy.

And $[\frac{d}{dE} \ln g]^{-1} = T$ the *Temperature.*

- Now suppose that we want to know the relative probability of 2 microstates of system 1 in equilibrium.
- There are, in all, g_1 of these states, for each specific E_1 but we want to know how many states of the combined system correspond to a single microstate of S_1.
- Obviously that is just equal to the number of states of system 2.
- So, denoting the two values of the energies of S_1 for the two microstates, we are comparing by E_A, E_B the ratio of the number of combined system states for S_{1A} and S_{1B} is

$$\frac{g_2(E_t - E_A)}{g_2(E_t - E_B)} = \exp[\sigma(E_t - E_A) - \sigma(E_t - E_B)] \qquad (14)$$

- Now we suppose that system S_2 is large compared with S_1 so that E_A and E_B represent very small changes in S_2's energy, and we can Taylor expand

$$\frac{g_2(E_t - E_A)}{g_2(E_t - E_A)} \simeq \exp\left[-E_A \frac{d\sigma}{dE} + E_B \frac{d\sigma}{dE}\right] \qquad (15)$$

- Thus we have shown that the ratio of the probability of a system (S_1) being in any two microstates A, B is simply

$$\exp\left[\frac{-(E_A - E_B)}{T}\right] , \qquad (16)$$

- Thus we have shown that the ratio of the probability of a system (S_1) being in any two microstates A, B is simply

- It may notice that Boltzmann’s constant is absent from this formula.
- That is because of using natural thermodynamic units for entropy (dimensionless) and temperature (energy).
- Boltzmann’s constant is simply a conversion factor between the natural units of temperature (energy, e.g. Joules) and (e.g.) degrees Kelvin.
- Kelvins are based on ◦C which arbitrarily choose melting and boiling points of water and divide into 100.
- Plasma physics is done almost always using energy units for temperature.

- Because Joules are very large, usually electron volts (eV) are used.

$$1eV = 11600K = 1.6 \times 10^{-19}\text{Joules.} \qquad (17)$$

- One consequence of our Botzmann factor is that a gas of moving particles whose energy is $\frac{1}{2}mv^2$ adopts the Maxwell Boltzmann (Maxwellian) distribution of velocities .

velocities $\propto exp\left[-\frac{mv^2}{2T}\right]$