## Group



**Definition:- Order (or period) of an element of group:-** If $G$ be a group and the composition be denoted multiplicatively, then an element $a$ in a group $G$ is said to be of order $n$ if $a^n = e$, the identity element and $n$ is the least positive integer.

If however for no positive integral value of $n$, $a^n = e$ then $a$ is said to be of ~~zero~~ order zero or infinity.

The notation for the order of $a$ is $o(a)$.

**Note:-** (i) Identity element $e$ is the only element of order one in a group.

(ii) In additive notation we use the words $na = e$ in place of $a^n$.

**Example:-** In the group $(a, a^2, a^3, a^4, a^5, a^6, a^7, a^8 = e)$ we find the order of $a$ is 8, since $a^8 = e$, the identity element. The order of $a^2$ is 4 then 4 is the least positive integer such that $(a^2)^4 = e$. Similarly the order of $a^4$ is 2 as $(a^4)^2 = a^8 = e$ and order of $a^3$ is zero as no positive integral power $a^3$ is equal to $a^8$ i.e. $e$ the identity element.

In a similar way in the infinite group

$(\cdots, a^{-4}, a^{-3}, a^{-2}, a^{-1}, a^{0} = e, a, a^{2}, a^{3}, a^{4}, \cdots)$

the order of every element is zero or infinite.

Thus we observe that if there exists a positive integer $m$ such that $a^m = e$, then the order of $a$ is finite. Also we must have $o(a) \leq m$.

If $a^m = e$, then the order of $a$ can be never be greater than $m$, at the most is equal to $m$. If $m$ itself is the least positive integer such that $a^m = e$, then we will have $o(a) = m$.

<u>Note 3:-</u> The order of every element of a finite group is finite whereas some or all of the elements of infinite group may be of finite order.

<u>Note 4:-</u> Order of identity element of every group (finite or infinite) is one

we have $e' = e \Rightarrow o(e) = 1$.

Also if $o(a) = 1$, then

$a' = a = e$

<u>Note 5 :-</u> In the additive group of integers only 0 has order one and no other element has finite order i.e.

all other elements are of infinite order.

Torsion group :-

Definition :- If every element of a group G is of finite order, then the group G is called torsion group or a periodic group.

The example of such groups are $(\{1, -1, i, -i\}, \cdot)$ and $(\{1, \omega, \omega^2\}, \cdot)$

Torsion free group :-

Definition :- If the order of every element except the identity element of a group G is not finite, then the group G is called torsion free group.

The examples of such group are $(I, +)$, $(R, +)$.

Mixed Group :-

Definition :- ~~If the~~ A group G is called a mixed group if there exist at least two elements (different from identity element) such that one of them is of finite order and the other of infinite order.