Exp. The table gives values of $\tan x$ for $0.10 \le x \le 0.30$

x	0.10	0.15	0.20	0.25	0.30
$\tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

find (a) $\tan(0.12)$ and $\tan(0.26)$

Sol. The finite difference table.

x	$\tan x$	Δ	Δ^2	Δ^3	Δ^4
0.10	0.1003 y_0	0.0508 Δy_0	0.0008 $\Delta^2 y_0$	0.0002 $\Delta^3 y_0$	0.0002 $\Delta^4 y_0$
0.15	0.1511	0.0516	0.0010	0.0004 $\nabla^3 y_n$	[illegible]
0.20	0.2027	0.0526	0.0014 $\nabla^2 y_n$		
0.25	0.2533	0.0540 ∇y_n			
0.30	0.3093 y_n				

(a) To find $\tan(0.12)$ $x = 0.12$ value lies in [illegible]

then $p = \frac{x - x_0}{h} = \frac{0.12 - 0.10}{.05} = \frac{0.02}{0.05} \Rightarrow p = 0.4$

(a) for $\tan(0.12)$ interpolation using Newton forward difference table values

$$\tan(0.12) = 0.1003 + 0.4(0.0508) + \frac{0.4(0.4-1)}{2} \times (0.0008) + \frac{0.4(0.4-1)(0.4-2)}{6}(0.0002) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{24}(0.0002)$$

$\tan(0.12) = 0.1205$

(b) To find $\tan(0.26)$ for $x = 0.26$ is near to 0.30 then use Newton Backward difference interpolation

$$p = \frac{x - x_n}{h} = \frac{0.26 - 3.0}{0.05} = \frac{-0.94}{0.05} = -0.8$$

$$\tan(0.26) = 0.3093 - 0.8(0.0540) + \frac{-0.8(-0.8+1)}{2} \times (0.0014) + \frac{-0.8(-0.8+1)(-0.8+2)}{6}(0.0004) + \frac{-0.8(-0.8+1)(-0.8+2)(-0.8+3)}{24}(0.0002)$$

$$\tan(0.26) = 0.2662$$

Hence, Newton's forward and backward interpolation formulae, which are applicable for interpolation near the beginning and end respectively.

Exp. Value of x (in degrees) and $\sin x$ are given in the following table

x (in deg)	15	20	25	30	35	40
$\sin x$	0.2588190	0.34…	0.42…	0.5	0.57…	0.64…

Determine the value of $\sin 38°$.

Sol. The difference Table

x	$\sin x$	Δ	Δ^2	Δ^3	Δ^4
15	0.2588190				
		0.0832011			
20	0.3420201		−0.0026029		
		0.0805982		−0.0006136	
25	0.4226183		−0.0032165		[illegible]
		0.0773817		−0.0005888	
30	0.5000000		−0.0038053		[illegible]
		0.0735764		−0.0005599 $\nabla^3 y_n$	
35	0.5735764		−0.0043652 $\nabla^2 y_n$		
		0.0692112 ∇y_n			
40	0.6427876 y_n				

To find $\sin 38°$, we use Newton's backward difference Interpolation formula with $x_n = 40°$ and $x = 38°$.

Thus $p = \dfrac{x - x_n}{h} = \dfrac{38° - 40°}{5} = \dfrac{-2}{5} = -0.4$

Hence, using Newton's backward difference interpolation formula

$$y_n(x) = y_n + p\nabla y_n + \frac{1}{2!}p(p+1)\nabla^2 y_n + \frac{1}{3!}p(p+1)(p+2)\nabla^3 y_n + \frac{1}{4!}p(p+1)(p+2)(p+3)\nabla^4 y_n + \frac{1}{5!}p(p+1)\cdots(p+4)\nabla^5 y_n$$

$$y(38) = 0.64278761 - \frac{0.4}{1!}(0.0692112) + \frac{-0.4(-0.4+1)}{2!}(-0.0042652) + \frac{-0.4(-0.4+1)(-0.4+2)}{3!}(-0.000[illegible])$$

$$+ \frac{-0.4(-0.4+1)(-0.4+2)(-0.4+3)}{4!}(0.0000289) +$$

$$\frac{-0.4(-0.4+1)(-0.4+2)(-0.4+3)(-0.4+4)}{5!}(0.00000[illegible])$$

$$= 0.6427876 - 0.02768440 + 0.00052382 +$$

$$0.00003583 - 0.00000120$$

$$= 0.6156614$$

$\sin(38°) = 0.6156614$

* Difference formulae can easily be establised by symbolic methods, using the shift operator (E) and the mean operator (μ).

(i) Mean operator (μ) is defined as

$$\mu y_r = \frac{1}{2}\left(y_{(r+1/2)} + y_{(r-1/2)}\right)$$

(ii) Shift operator (E) is defined by as

$$E y_r = y_{r+1} \quad (E = 1 + \Delta)$$

$$E^2 y_r = E(E y_r) = E y_{r+1} = y_{r+2}$$

which shows that the effect of E is to shift the functional value of y_r to the next higher value y_{r+1}. In general $E^n y_r = y_{r+n}$

$$\Delta y_0 = y_1 - y_0 = E y_0 - y_0 = (E-1) y_0 \Rightarrow (E = \Delta + 1)$$