*Backward difference operator(∇):

The backward difference operator ∇ defined by-

$$\nabla f(x) = f(x) - f(x-h)$$

or $\nabla y_i = y_i - y_{i-1}$ $\quad i=1,2,\ldots\ldots n$

$$\nabla y_1 = y_1 - y_0$$

$$\nabla y_2 = y_2 - y_1$$

.

.

$$\nabla y_n = y_n - y_{n-1}$$

Similer way can define backward differences of higher orders Thus

$$\nabla^2 y_2 = \nabla(\nabla y_2)$$

$$= \nabla(y_2 - y_1) \Rightarrow \nabla y_2 - \nabla y_1$$

$$= y_2 - y_1 - y_1 + y_0$$

$$\nabla^2 y_2 = y_2 - 2y_1 + y_0$$

$$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$$

$$= y_3 - 3y_2 + 3y_1 - y_0 \quad \text{etc}$$

With the same values of x and y in forward difference table.

Backward Difference Table

x	y	∇	∇^2	∇^3	∇^4	∇^5	∇^6
x_0	y_0						
x_1	y_1	∇y_1					
x_2	y_2	∇y_2	$\nabla^2 y_2$				
x_3	y_3	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$			
x_4	y_4	∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_4$		
x_5	y_5	∇y_5	$\nabla^2 y_5$	$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$	
x_6	y_6	∇y_6	$\nabla^2 y_6$	$\nabla^3 y_6$	$\nabla^4 y_6$	$\nabla^5 y_6$	$\nabla^6 y_6$

Central Difference operator (δ):-

The central difference operator δ is defined by the relations as $\delta f(x) = f(x+h/2) - f(x-h/2)$

$$y_1 - y_0 = \delta y_{1/2}$$

$$y_2 - y_1 = \delta y_{3/2}$$

$$y_3 - y_2 = \delta y_{5/2}$$

$$y_n - y_{n-1} = \delta y_{n-1/2}$$

Similarly, higher order central differences can be defined.

Central Difference Table.

x	y	δ	δ^2	δ^3	δ^4	δ^5	δ^6
x_0	y_0						
		$\delta y_{1/2}$					
x_1	y_1		$\delta^2 y_1$				
		$\delta y_{3/2}$		$\delta^3 y_{3/2}$			
x_2	y_2		$\delta^2 y_2$		$\delta^4 y_2$		
		$\delta y_{5/2}$		$\delta^3 y_{5/2}$		$\delta^5 y_{5/2}$	
x_3	y_3		$\delta^2 y_3$		$\delta^4 y_3$		$\delta^6 y_3$
		$\delta y_{7/2}$		$\delta^3 y_{7/2}$		$\delta^5 y_{7/2}$	
x_4	y_4		$\delta^2 y_4$		$\delta^4 y_4$		
		$\delta y_{9/2}$		$\delta^3 y_{9/2}$			
x_5	y_5		$\delta^2 y_5$				
		$\delta y_{11/2}$					
x_6	y_6						

It is clear from three tables forward, backward and central differences that in definite numerical case, the same numbers occur in the same positions weather we use forward (Δ), Backward (∇) or central (δ) differences.

Thus.

$$\Delta y_0 = \nabla y_1 = \delta y_{1/2}$$

Thus, forward difference (FD, Δ), backward difference (BD, ∇) and central difference (CD, δ) are used for interpolation value of x from given the table values (x_i, y_i), $i = 0, 1, 2, \dots n$ of any function $y = f(x)$, $x_0 \le x \le x_n$ &

$$x_i = x_0 + ih$$

FD $\Rightarrow$ $\Delta f(x) = f(x+h) - f(x)$ or $\Delta y_i = y_{i+1} - y_i$

BD $\Rightarrow$ $\nabla f(x) = f(x) - f(x-h)$ or $\nabla y_{i+1} = y_{i+1} - y_i$

CD $\Rightarrow$ $\delta f(x) = f(x + \frac{h}{2}) - f(x - h/2)$ or

$$\delta y_i = y_{(i+\frac{1}{2})} - y_{(i-\frac{1}{2})}$$

Exp. for a Table

Forward

x	2	4	6	8	$h=2$
y	10	20	30	40	

Backward

In this table, we want to interpolate value at $x = 3$, $x = 7$ & $x = 5$ then which difference operator is suitable for interpolating value according to x.

1) for $x = 3$, it is near to 2 & 4 then use forward difference. It is suitable for interpolating value of x.

(ii) For $x=7$, this value is near to 6 & 8 then use Backword difference.

(iii) For $x=5$ lies in mid of 4 & 6 then we should use central difference.

So, first order differences for this Table

x	$y=f(x)$	$\Delta f(x)$	$\nabla f(x)$	$\delta f(x)$
$x_0=2$	$y_0=10$			
		$\Delta y_0=y_1-y_0$,	$\nabla y_1=y_1-y_0$,	$\delta y_{1/2}$
$x_1=4$	$y_1=20$			
		$\Delta y_1=y_2-y_1$,	$\nabla y_2=y_2-y_1$,	$\delta y_{3/2}$
$x_2=6$	$y_2=30$			
		$\Delta y_2=y_3-y_1$,	$\nabla y_3=y_3-y_2$,	$\delta y_{5/2}$
$x_3=8$	$y_3=40$			

Hence

$$\Delta y_0 = y_1 - y_0 = \nabla y_1 = \delta y_{1/2} = 10$$

This relation shows that the first order of forword difference, Backward difference and central difference value y_1-y_0 is 10 i.e. $\Delta y_0=\nabla y_1=\delta y_{1/2}=10$ equal.