Numerical Analysis for UG - Part-III

Numerical analysis is to provide efficient methods for obtaining numerical solutions to such problems arising in different areas of higher mathematics.

The main concern of analysis of numerical methods is to provide computer-oriented, efficient and reliable numerical methods for solving problems in research and industry.

Therefore numerical methods are used to solving the following mathematical problems in different areas.

(I) **Algebraic and transcendental (non-Algebraic) Equations:-** To solving the problem of nonlinear Equations of type $f(x)=0$ is frequently encountered in engineering. Exp. A nonlinear Equation for t.

$$\frac{m_0}{m_0 - u_f t} = e^{(u+gt)/u_0}$$

when m_0, g, u, u_0 and u_f are given. This type Equation occur in rocket studies.

) **Interpolation:-** A function $y = f(x)$ and given of data values (x_i, y_i), $i = 0, 1, 2, \ldots n$. To find the value of y for a given value of x where $x_0 < x < x_n$.

This process is called interpolation.

If this process is used for functions of several variables, it is called multivariate interpolation.

Exp. $y = mx + c$, where m & c are constants for this &

(iii) Curve fitting - Where data points are subject to errors both round off and systematic. Curve fitting is useful when interpolation formulae yield unsatisfactory solutions. In such cases, the method is to fit a curve which passes through the data points and then use the curve to predict the intermediate values. Curve fitting is usually in data smoothing. for Exp. Incoming Solar rad. radiation at surface (hourly data) may be fluctuate by aerosols and clouds. This data can be smooth by using curve fitting analysis method.

(iv) Numerical differentiation and integration
It is often required to determine the numerical values of

(i) $\frac{dy}{dx}, \frac{d^2y}{dx^2},$ for a certain value of x in $x_0 \le x \le x_n$

(ii) $I = \int_{x_0}^{x_n} y\,dx$

where the set of data values (x_i, y_i), $i = 0, 1, \ldots n$ is given, but the explicit nature of $y(x)$ is not known.

Exp.- If the data consist of angle θ (in radians) of a rotating rod for values of time t (in seconds), then its angular velocity and angular acceleration at any time can be computed by numerical differentiation method.

These numerical methods can be solved by manually and computer programmings. It is well known that mathematics and computer programming languages are two important tools of numerical methods. Many programming languages are used to solving the numerical methods. There are limitations of every language. Several programming languages are as.

(I) FORTRAN→(Formula Translation) was introduced by IBM in 1957. It is readily available on almost all computers and one of its important features is that it allows a programmer to express the mathematical algorithm more precisely.

II) BASIC→ Originally developed by John kemeny and Thomas Kurtz in 1960. BASIC was used in starting few years only for instructions process of purposes. At present version of BAS is Visual Basic. It is easy to use.

III) C languange ÷ This is a high-level programming language developed by Bell Teleph laboratories in 1972. It is case sensitive language. C is a powerful general purpose programming language. It can be solved numerical methods and useful to develop Software.

Algebraic functions of the form
$f_n(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n$,
are called polynomials.

A non-algebraic function is called transcendental function, e.g. $f(x) = \ln x^3 - 0.7$
$\phi(x) = e^{-0.5x} - 5x$
$\psi(x) = \sin^2 x - x^2 - 2$

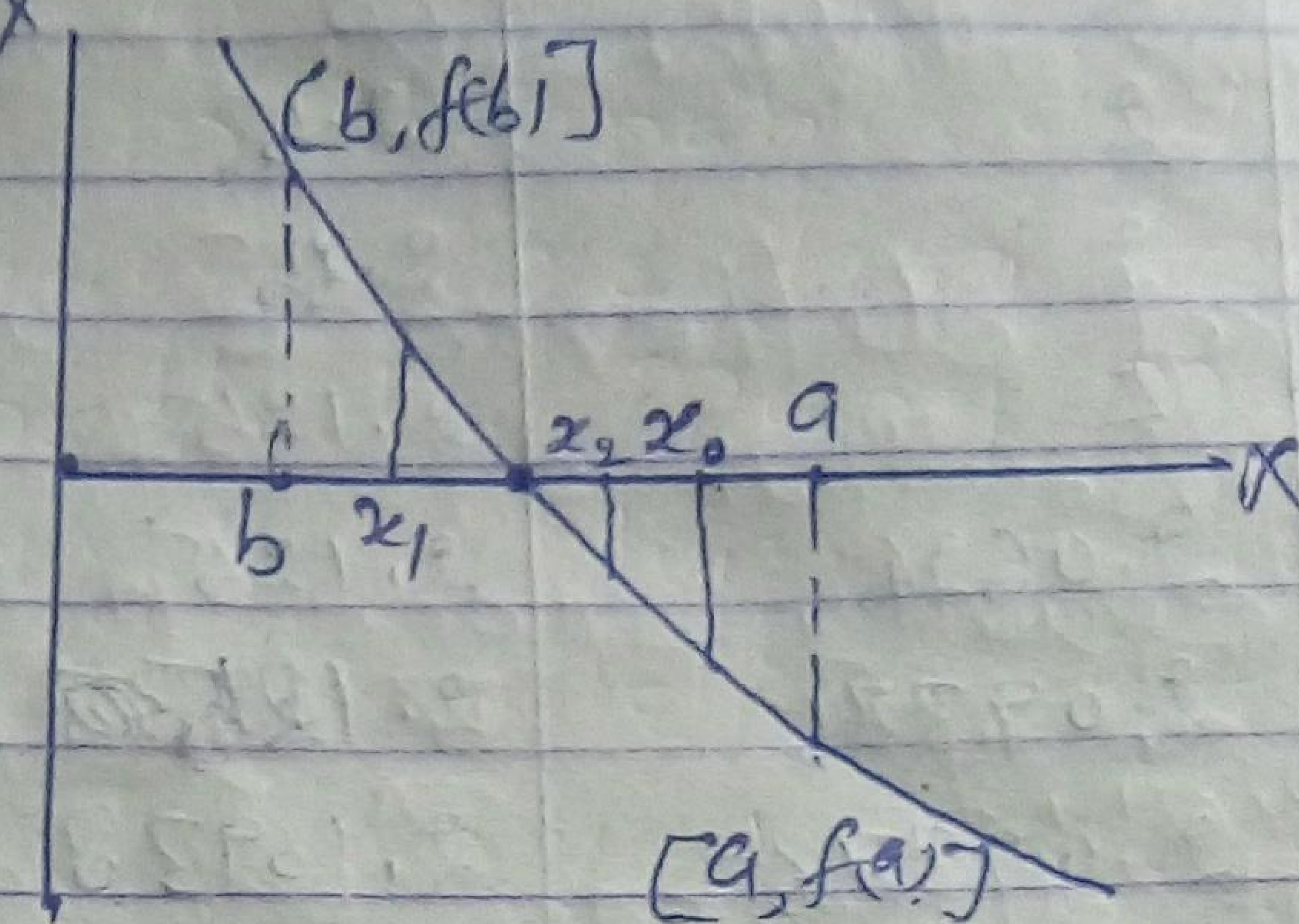
We can use numerical methods for finding a real root of algebraic or non-algebraic (transcendental) equations and also methods of determining all real and complex roots of polynomials.

The Bisection Method:- If a function $f(x)$ is continuous between a and b, and $f(a)$ and $f(b)$ are of opposite signs, then there exists at least one root between a and b.
Let $f(a)$ be negative and $f(b)$ be positive then the root lies between a and b

$$x_0 = (a+b)/2$$

If $f(x_0) = 0$ then x_0 is a root of the equation $f(x) = 0$.
Otherwise, the root lies either between x_0 and b or x_0 and a depending on $f(x_0)$ negative or positive sign.

* Graphical representation of the Bisection method. →



This method always succeeds. If there are more roots than one in the interval, bisection method finds one of the roots.

Exp. find a real root of the equation:

$$x^3 - 2x - 5 = 0$$

Let $f(x) = x^3 - 2x - 5$ Then

$f(2) = (2)^3 - 2\times2 - 5 = 8 - 9 = -1$

$f(3) = (3)^3 - 2\times3 - 5 = 27 - 6 - 5 = 16$

Hence a root lies between 2 and 3 and we take

$$x_0 = \frac{2+3}{2} = 2.5$$

Since $f(x_0) = (2.5)^3 - 2\times2.5 - 5 = 15.625 - 10$

$f(x_0) = 5.6250$, Now new interval $[2, 2.5]$

Then $x_1 = \frac{2+2.5}{2} = 2.25$

and

$f(x_1) = 1.890625$

Proceeding in this way, the following table is obtained

Table:- For Bisection method.

n	a	b	$x=(a+b)/2$	$f(x)$
1	2	3	2.5	5.6250
2	2	2.5	2.25	1.8906
3	2	2.25	2.125	0.3457
4	2	2.125	2.0625	-0.3513
5	2.0625	2.125	2.09375	-0.0089
6	2.09375	2.125	2.10938	0.1668
7	2.09375	2.10938	2.10156	0.07856
8	2.09375	2.10156	2.09766	0.03471
9	2.09375	2.09766	2.09570	0.01286
10	2.09375	2.09570	2.09473	0.00195
11	2.09375	2.09473	2.09429	-0.0035
12	2.09429	2.09473		

At n=12, it is seen that the difference between two successive iterates is 0.0005, which is less than 0.001. So correct value of ~~x~~ near to $x=2.09473$

* Regula falsi or method of chords. The Equation of the chord ~~joing~~ joining the two points $[a, f(a)]$ and $[b, f(b)]$ is given by this equation

$$\frac{y-f(a)}{x-a} = \frac{f(b)-f(a)}{b-a}$$

The method consists in replacing the part of the curve between the points $[a, f(a)]$ and $[b, f(b)]$ by means of the chord joining these points. The point of intersection in this case by putting $y=0$ in above equation.