Newton-Raphson Method :-

Let x_0 be an approximate root of the equation $f(x)=0$ if $x_1 = x_0+h$ be the exact root, then $f(x_1)=0$

$\therefore$ Expanding $f(x_0+h)$ by Taylor's series

$$f(x_0)+hf'(x_0)+\frac{h^2}{\lfloor 2}f''(x_0)+\cdots\cdots=0$$

Since h is small, neglating h^2 and higher power of h, we get

$$f(x_0)+hf'(x_0)=0$$

$$h=-\frac{f(x_0)}{f'(x_0)} \quad \text{——} \quad (1)$$

$\therefore$ A closer approximation to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly, starting with x_1, a still better approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{——} \quad (2)$$

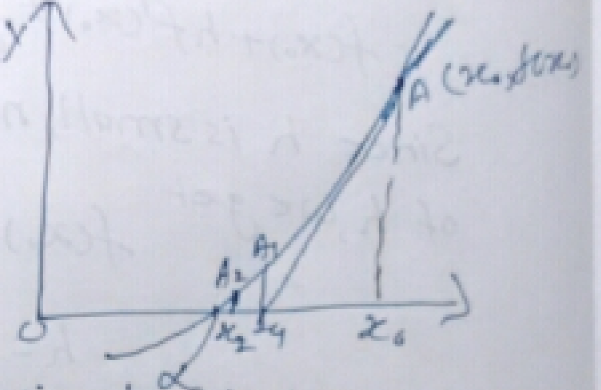
Which is known as the Newton-Raphson formula or Newton's Iteration formula.

Geometrical Interpretation:-

Let x_0 be a point near the root α of the equation $f(x)=0$ (in fig). Then the equation of the tangent at $A_0(x_0, f(x_0))$ is —

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$



Which is a first approximation to the root α. If A_1 is the point corresponding to x_1 on the curve then the tangent at A_1 will cut the x-axis of x_2 which is near to α and is, therefore, a second approximation to the root. Repeating the process we approach to the end root α quite rapidly. Hence the method consists in replacing the part of the curve between the point A_0 and the x-axis by means of the tangent to the curve at A_0.