Zone Plate

What is Zone Plate? Give the Construction and theory of a Zone Plate. show that zone plate has multiple focii. compare the zone plate with a Convex lens.

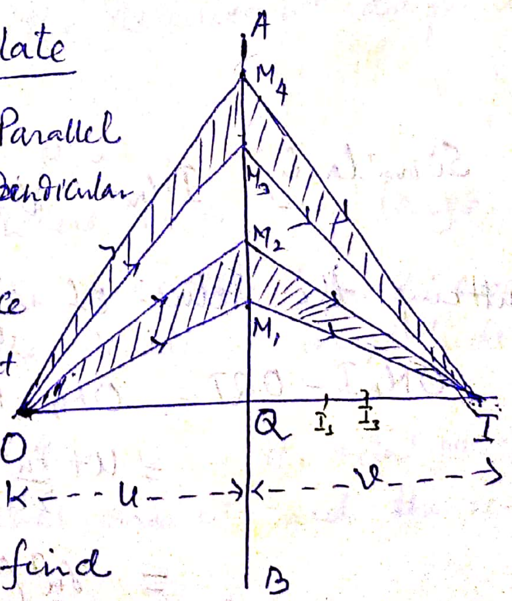
ANS. A zone plate is a special diffracting screen based on Fresnel's theory of half-Period zones and it is so constructed that alternate zones obstruct the light.

It is constructed by drawing a series of concentric circles on a sheet of Papers with radii Proportional to the square root of the natural numbers like 1, 2, 3, 4 .. etc.

The alternate zones (either even or odd order zones) are covered with black ink and a highly reduced photograph of the drawing is taken on a plane glass plate of uniform thickness. The negative thus obtained is a zone plate. If the central half period zone is clear (a) it is called Positive zone plate. If the central zone is opaque (b) it is called a negative zone plate.

Theory of Zone Plate

Let AB be a plane parallel transparent screen perpendicular to the plane of paper and O be point source of monochromatic light of wavelength λ.



We have to find the resultant amplitude at I due to secondary wavelets from the various points of the screen under the influence of spherical waves diverging from O.

Let us divide the screen into half period zones by chosing point $M_1, M_2, M_3 \cdots M_n$ on the screen such

$$OM_1I - OQI = \lambda/2$$

$$OM_2I - OQI = 2\cdot\lambda/2$$

$$OM_3I - OQI = 3\cdot\lambda/2$$

$$\cdots\cdots\cdots$$

$$OM_nI - OQI = n\lambda/2 \quad \text{——— (1)}$$

(According to Freshel paths of the waves reaching I through two consecutive zones differ by $\lambda/2$)

If concentric circles be drawn on the screen with Q as centre and radii equal to $QM_1 = r_1$, $QM_2 = r_2$ ... $QM_n = r_n$, the area of the first circle is first half period zone, and the area between 2nd circle & first circle is the second half period zone and so on.

Let $OQ = u$ and $QI = v$ then

$$OM_n^2 = QM_n^2 + OQ^2 = u^2 + r_n^2 \quad \text{where } QM_n = r_n.$$

$$\therefore OM_n = (u^2 + r_n^2)^{1/2} = u\left[1 + \frac{r_n^2}{u^2}\right]^{1/2}$$

$$= u + \frac{r_n^2}{2u} \quad \text{---- (2)}$$

(to a first approximation)

Similarly $IM_n = v + \frac{r_n^2}{2v}$ --- (3)

Putting the value of eqn. (2) & (3) in (1), we get

$$OM_nI - OQI = OM_n + M_nI - OQI = n\lambda/2$$

$$= u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} - (u+v)$$

$$= \frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) = n\lambda/2$$

$$\text{Or } r_n^2 = \frac{uvn\lambda}{u+v} \quad \text{——— (4)}$$

$$\therefore \; r_n = \left(\frac{uv\lambda}{u+v}\right)^{\frac{1}{2}} \sqrt{n}$$

ie. $r_n \propto \sqrt{n}$ ie the radii of circles are Proportional to square root of natural numbers.

The area of the nth half Period zone is

$$\pi r_n^2 - \pi r_{n-1}^2 = \pi \left\{ \frac{uvn\lambda}{u+v} - \frac{uv(n-1)\lambda}{u+v} \right\}$$

$$= \frac{\pi uv\lambda}{u+v} \quad \text{—(5)}$$

From this eqn (5) it is evident that the area of all the zones are equal. If u becomes infinite we shall have Plane waves falling on the screen. The area of each half Period zone reduces to $\pi v \lambda$

Let $R_1, R_2, R_3 \ldots$ be the amplitudes at I due to Secondary wavelets from 1st, 2nd, 3rd .... zones respectively. Due to obliquity factor $(1+\cos\theta_n)$ the magnitude of $R_1, R_2, R_3 \cdots$ are in decreasing order with the increase of n. Now the amplitudes from alternate zones will have the opposite Phases. So, the resultant amplitude at I is given by

$$R = R_1 - R_2 + R_3 - R_4 + \cdots$$
$$= \frac{1}{2}R_1 \text{ where } n \text{ is very large.}$$

If we intercept the wavelets from the even numbered zones, the Resultant amplitude at I is $R' = R_1 + R_3 + R_5 + \cdots$

$$= \frac{1}{2} n R_1 \text{ where } n \text{ is the total No. of Zones.}$$

Again if odd zones are blocked, the resultant amplitude at I is

$$R' = R_2 + R_4 + R_6 + \cdots$$

Thus we see that in both cases the resultant amplitude at I is many times greater than that of entire zones.

In other words we say that I will be the image of O. From eq. (4), the relation between u & v is given by.

$$\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2} \quad -(6)$$

If O lies at infinity and v = f, the focal length

Hence $\frac{1}{f} = \frac{n\lambda}{r_n^2}$ or $f = \frac{r_n^2}{n\lambda}$

This result is similar to that for a lens. Thus with regard to light from a luminous point on the axis of a zone plate, it acts like a lens.

Comparison with a Convex lens

Similarities

(I) Both form real image of an object on the side opposite to that of the object.

(II) The relelation between the conjugate distance are similar for both.

(III) Both show chromatic aberration because the focal length varies with λ for both.

Dissimilarities

(I) For a given wavelength of light, a convex lens has a fixed focal length given by $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

But a zone plate for the same type of light has multiple focal lengths and therefore forms a series of images (or focii) of decreasing intensities for a point source because the no. of half period zones contained in an area depend upon the position of the screen (i.e. V), and for each such positions, there is an odd no. of half period element.

(II) For a convex lens all the rays reaching the image point have the same optical path but for a zone plate the rays reaching the image-point through two successive transparent zones have a path diff. equal to λ.

(III) In a zone plate each colour has its own seperate focus, the red being nearer the plate than the violet

Scanned with CamScanner

Since $f = r_n^2/n\lambda$ and $\lambda_r > \lambda_v$. Hence $f_r < f_v$.
In case of a ~~lens~~ Convex lens, however it is just the reverse, red being less refrangible than violet comes to focus farther away from the lens i.e. $f_r > f_v$

(IV) The zone plate also acts ~~as~~ a simultaneously as a concave lens unlike the lens it has multiple virtual focii on the side of the source.

(V) The image due to a zone plate is less intense than that due to convex lens.

Multiple focii of a zone plate

A zone plate has a no. of focii which depends upon the number of zones used as well as the wavelength of the light used. For $u = \infty$, $f = v$ and the radius of the nth circle will be given $r_n^2 = vn\lambda$ (Putting $u = \infty$ in (5)).

And area of nth half period zone $= \pi v \lambda$.
Hence in this case light will be brought to focus at a point P at a distance $v = r_n^2/n\lambda$, for which area of each zone is $\pi v \lambda$. The point given by this relation is known as primary focus or the first order focal ~~length~~ point and is the most intense point.

There is a series of focii unlike a convergent lens, of diminishing intensities as we go along the axis towards the zone plate. The absence of these focii is simply due to the reason that the area of each zone diminishes as points nearer to the plate are concerned.

Let us therefore consider a point I_3 at a distance $v = r_n^2/3n\lambda$ from the zone plate. If the zone plate be imagined to be divided into half period elements, the area of each element will be one third the area of each zone. Hence each zone will contain three half period elements corresponding to I_3. The 2nd black zone intercepts 4th, 5th, and 6th half period elements while the wavelets from 7th, 8th and 9th half period elements contained in the original third half-period zones are transmitted & so on.

Therefore the resultant amplitude at I_3 will be

$$S = (S_1 - S_2 + S_3) + (S_7 - S_8 + S_9) + (S_{13} - S_{14} + S_{15}) + \cdots$$

$$= \left\{ S_1 - \tfrac{1}{2}(S_1 + S_3) + S_3 \right\} + \left\{ S_7 - \tfrac{1}{2}(S_7 + S_9) + S_9 \right\} + \cdots$$

$$= \tfrac{1}{2}(S_1 + S_3 + S_7 + \cdots) \text{ approx.}$$

where $S_1, S_2, S_3 \cdots$ are roughly one third of $R, R_2 - R_1$ etc. Hence Point I_3 is of Max^m. intensity which is ~~analogous~~ therefore, the second Point but it is much less intense than I. The light rays transmitted through each successive transparent zone have a Path diff. of 3λ. Hence I_3 is also called the third order focal point.

Similarly it can be shown that Points $I_5, I_7 \cdots$ etc distant $\frac{r_n^2}{5n\lambda}, \frac{r_n^2}{7n\lambda} \cdots$ from the zone plate are a series of images of the Point object O, but of successively diminishing intensities. Hence we conclude that a zone plate has multiple foci. unlike a convex lens, the various focal lengths will be $\frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda} \cdots$ etc.