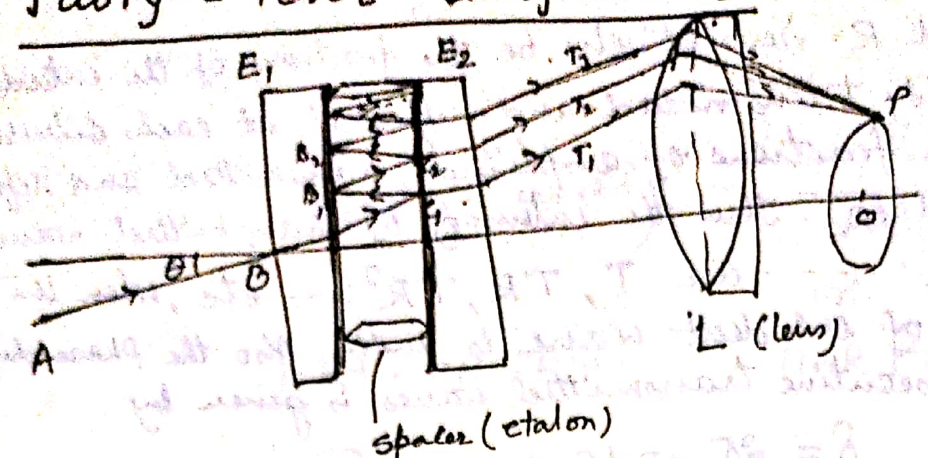
Fabry - Perot Interferometer



<u>Construction</u>: A Fabry - Perot interferometer consists of two glass plates forming a parallel air film between them. The faces of two plates in front of each other which have a parallel air film between them are lightly silvered. The outer surfaces are plane and wedge shaped. It is done so to avoid interference between the rays reflected at the outer unsilvered surface. One plate is fixed and other is attached to a carriage which can be moved towards or away from fixed plate and the shift can be measured accurately by scale.

<u>Principle</u>: The instrument works on the principle of interference produced by multiple reflection in the air film between two plates lightly silvered

<u>working</u>: As shown in the above fig, Let a monochromatic light from a broad source be incident on the plate E_1 at an angle θ. As a result of multiple internal reflections and consequent transmission we obtain a set of infinite parallel transmitted waves like C_1T_1, C_2T_2 ... etc. since these waves are obtained from the same incident wave, they interfere when combined in the 2nd focal plane of lens L.

The interference fringes are circular in nature. The condition for maxima is $2d\cos\theta = n\lambda$
where d = spacing between parallel plates
& n = order of fringes.

The interference pattern consists of a system of bright concentric rings on a dark background, each ring corresponds to a particular value of θ.

(2)

Intensity distribution:

Let T and R respectively be the fractions of the incident light intensity transmitted and reflected at each silvered surface; The fractions of amplitude transmitted and reflected are $\sqrt{T}$ and $\sqrt{R}$. Then the intensity of transmitted waves C_1T_1, C_2T_2, ... are T, TR, TR^2... etc, when the amplitude of incident wave is unity. Also the phase difference between consecutive transmitted waves is given by

$$\delta = \frac{2\pi}{\lambda} \times 2d\cos\theta \quad \text{—— (11)}$$

Therefore, if the incident wave is represented by $y = \sin\omega t$, the successive transmitted waves can be represented by $y_1 = T\sin\omega t$, $y_2 = TR\sin(\omega t - \delta)$

$y_3 = TR^2\sin(\omega t - 2\delta)$ ... and so on.

The negative signs are there because phase angles decreases as the path difference increases.

The final resultant vibration at P can be written as $D\sin(\omega t - \phi)$ where D is the instaneous resultant amplitude and ϕ is the resultant phase. Now by principle of superposition of wave motions, we have

$$D\sin(\omega t - \phi) = T\sin\omega t + TR\sin(\omega t - \delta) + TR^2\sin(\omega t - 2\delta) + \cdots$$

Expanding sine terms and equating the co-efficients of $\sin\omega t$ and $\cos\omega t$ on the two sides we may obtain

$$D\cos\phi = T + TR\cos\delta + TR^2\cos 2\delta + \cdots$$

and $$D\sin\phi = TR\sin\delta + TR^2\sin 2\delta + TR^3\sin 3\delta + \cdots$$

Hence the resultant intensity (I) is given by

$$I = D^2 = (D\cos\phi + iD\sin\phi)(D\cos\phi - iD\sin\phi)$$

where $i = \sqrt{-1}$

Here $$(D\cos\phi + iD\sin\phi) = T(1 + Re^{i\delta} + R^2e^{2i\delta} + \cdots)$$

$$= \left(\frac{T}{1 - Re^{i\delta}}\right)$$

and $\left(D\cos\phi - iD\sin\phi\right) = T\left(1 + Re^{-i\delta} + R^2e^{-2i\delta} + \cdots\right)$

$$= \frac{T}{1-Re^{-i\delta}}$$

Hence $I = \dfrac{T^2}{(1-Re^{i\delta})(1-Re^{-i\delta})} = \dfrac{T^2}{1+R^2-2R\cos\delta}$

$$= \frac{T^2}{(1-R)^2+2R-2R\cos\delta} = \frac{T^2}{(1-R)^2+4R\sin^2\delta/2}$$

$$= \frac{T^2}{(1-R)^2}\left[\frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\delta/2}\right] \quad \text{---- (III)}$$

This expression gives the variation of resultant intensity with $\sin^2\delta/2$ and with the film properties.

For maximum intensity, $\sin^2\frac{\delta}{2}=0$ or $\delta = 2n\pi$

$$\therefore I_{max} = \frac{T^2}{(1-R)^2}$$

For minimum intensity; $\sin^2\delta/2 = 1$ or $\delta = (2n+1)\pi$

$$\therefore I_{min} = \frac{T^2}{(1+R)^2}$$

when there is no absorption at each reflecting surface

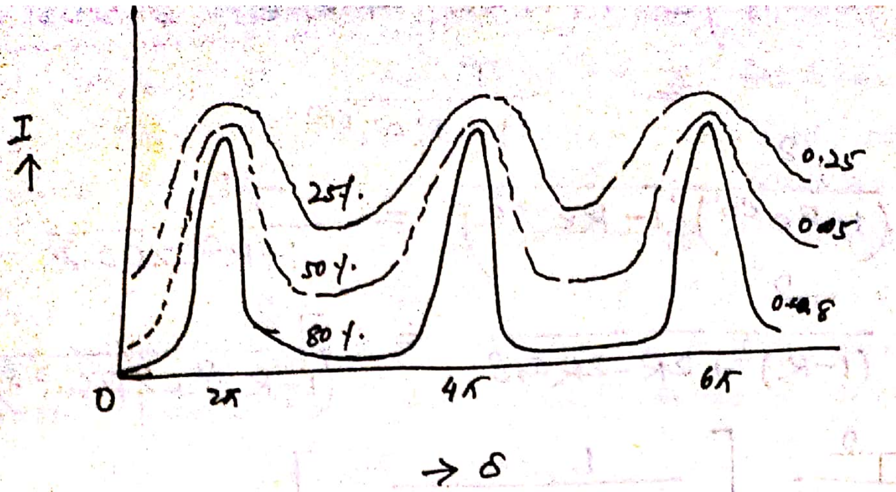
$$T+R=1 \quad \text{or} \quad T=(1-R)$$

and thus $I_{max} = \dfrac{T^2}{(1-R)^2} = 1$ and $I_{min} = \dfrac{(1-R)^2}{(1+R)^2}$

In general equation (III) can be written as

$$I = I_{max}\left\{\frac{1}{1+F\sin^2\delta/2}\right\} \quad \text{where } F = \frac{4R}{(1-R)^2}$$

<u>Sharpness of fringes</u>: The plot of I verses δ shows that intensity depends upon R, the reflectivity. When R is very ~~the~~ large ~~the quantity~~ the intensity falls more rapidly on either side of maximum and greater is the difference between Max^m & Min^m.



Further the visibility of the fringes is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{I_{max}}{I_{min}} - 1 \Big/ \frac{I_{max}}{I_{min}} + 1$$

$$= \frac{(1+R)^2}{(1-R)^2} - 1 \Big/ \frac{(1+R)^2}{(1-R)^2} + 1 = \frac{2R}{(1+R^2)}$$

Thus it follows that visibility of the fringes depend upon only the reflecting power of the surfaces and is independent of transmission co-efficient.

Determination of wavelength

The interferometer is first adjusted to form circular fringes in the centre of the field of view. Suppose the separation between the plates is such that a bright fringe of order n is obtained at the centre. In this condition

$$2d = n\lambda$$

It is clear that each time the movable plate moves through a distance $\lambda/2$ and the next bright fringe comes into field of view.

To determine λ, the movable plate is moved from the position x_1 to a position x_2 and the no. of bright fringes N that appears at the centre are counted. Then evidently

$$N \cdot \lambda/2 = x_2 - x_1$$

$$\therefore \lambda = \frac{2(x_2 - x_1)}{N}$$, From this relation λ is computed.