Iterative Methods of Solution:-

The preceding methods of solving simultaneous linear equations are unknown as direct methods as they yield exact solutions. On the other hand, an iterative methods is that in which we start from an approximation to the true solution and obtain better and better approximations from a computation cycle repeated as often as ~~many~~ may be necessary for achieving a desired accuracy.

Simple iterative methods can be derived for systems in which the coefficients of the leading diagonal are large compared to others. These are three methods.

(I) Jacobi's Iterative Method
(II) Gauss-Seidal Iterative Method
(III) Relaxtion Method.

We now explain Jacobi's Iterative method.

Jacobi's Iterative Method:-

Consider the equations.

$$\left.\begin{aligned} a_1x+b_1y+c_1z &= d_1 \\ a_2x+b_2y+c_2z &= d_2 \\ a_3x+b_3y+c_3z &= d_3 \end{aligned}\right\} \quad \text{---(1)}$$

If a_1, b_2, c_3 are large as compared to other coefficients, then solving these for x, y, z respectively, the system can be written in the form:-

$$\left.\begin{aligned} x &= k_1 - l_1 y - m_1 z \\ y &= k_2 - l_2 x - m_2 z \\ z &= k_3 - l_3 x - m_3 y \end{aligned}\right\} \quad \text{——— (2)}$$

Let us start with the initial approximations x_0, y_0, z_0 (each $= 0$) for the values of x, y, z.
Substituting these on the right sides of Eqn (2) then we get the first approximations

$$x_1 = k_1 \quad , \quad y_1 = k_2 \quad \text{and} \quad z_1 = k_3$$

Substituting these on righ-hand sides of Eqn (2) the second approximations are given by

$$x_2 = k_1 - l_1 y_1 - m_1 z_1$$
$$y_2 = k_2 - l_2 x_1 - m_1 z_1$$
$$z_2 = k_3 - l_3 x_1 - m_3 y_1$$

This process is repeated till the difference between two consecutive approximations is negligible.

Exp. Solve by Jacobi's Iterative method the equations.

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Solution: The diagonal coefficients of the given equations are large so applying Jacobi's Iterative method.

We write the given Equations in the form:-

$$\left.\begin{aligned} x &= \frac{1}{20}(17 - y + 2z) \\ y &= \frac{1}{20}(-18 - 3x + z) \\ z &= \frac{1}{20}(25 - 2x + 3y) \end{aligned}\right\} \quad \text{——— (1)}$$

We start from the approximation $x_0 = y_0 = z_0 = 0$ putting these on the right-hand sides of Eqn (1) we get

$$x_1 = \frac{17}{20} = 0.85$$

$$y_1 = \frac{-18}{20} = -0.9$$

$$z_1 = \frac{25}{20} = 1.25$$

Putting these values in the right-hand sides of equation (1) we obtain

$$x_2 = \frac{1}{20}(17 - y_1 + 2z_1) = \frac{1}{20}(17 + 0.9 + 2\times1.25)$$
$$= 1.02$$

$$y_2 = \frac{1}{20}(-18 - 3x_1 + z_1) = \frac{1}{20}(-18 - 3\times0.85 + 1.25) = -0.965$$

$$z_2 = \frac{1}{20}(25 - 2x_1 + 3y_1) = \frac{1}{20}(25 - 2\times0.85 + 3\times-0.90) = 1.1515$$

Substituting these values in the right sides of equation (1) we have

$$x_3 = \frac{1}{20}(17 - y_2 + 2z_2) = \frac{1}{20}(17 + 0.965 + 2\times1.1515)$$
$$= 1.0134$$

$$y_3 = \frac{1}{20}(-18 - 3x_2 + z_3) = \frac{1}{20}(-18 - 3\times1.02 + 1.1515)$$
$$= -0.9954$$

$$z_3 = \frac{1}{20}(25 - 2x_2 + 3y_3) = \frac{1}{20}(25 - 2\times1.02 + 3(-0.965))$$
$$= 1.0032$$

Substituting these value in Eqn (1) we get

$x_4 = \frac{1}{20}(17 - y_3 + 2z_3) = 1.009$

$y_4 = \frac{1}{20}(-18 - 3x_3 + z_3) = -1.0018$

$z_4 = \frac{1}{20}(25 - 2x_3 + 3y_3) = 0.9993$

Again substituting these value in Eqn ① we get

$x_5 = \frac{1}{20}(17 - y_4 + 2z_4) = 1.0000$

$y_5 = \frac{1}{20}(-18 - 3x_4 + z_4) = -1.0002$

$z_4 = \frac{1}{20}(25 - 2x_4 + 3z_4) = 0.9996$

Again substituting we get

$x_6 = \frac{1}{20}(17 - y_5 + 2z_3) = 1.0000$

$y_6 = \frac{1}{20}(-18 - 3x_5 + z_5) = -1.0000$

$z_6 = \frac{1}{20}(25 - 2x_5 + 3z_5) = 1.0000$

The values of the 5th & 6th approximations values being particular the same. Hence the solution is

$x = 1$

$y = -1$

$z = 1$