GROUP THEORY

Centre of a group :-

Definition :- Let (G, o) be a group then Centre of G is denoted as $Z(G)$ and is defined as

$$Z(G) = \{a, G : ax = xa \ \forall \ x \in G\}$$

Concept :-

(i) $Z(G) \neq \phi$ since $e \in Z(G)$

$\left[\because e \in G \right.$
$e o x = x o e = x, \forall \ x \in G$
e is the Centre of $G \left. \right]$

(ii) if G is abelian $\Rightarrow Z(G) = G$

Ex:- 1 We Consider the group $(G, \cdot)$. Find $Z(G)$

$$G = \{1, -1, i, -i\}$$

sol:- for $1 \in G$

$1 \cdot x = x \cdot 1$, $\forall \ x \in G \Rightarrow 1 \in Z(G)$

for -1

$-1 \cdot x = x(-1) = -x \quad \forall \ x \in G \Rightarrow -1 \in Z(G)$

$i \in Z(G)$, $-i \in Z(G)$

$Z(G) = G = \{1, -1, i, -i\}$. Each element Commute with every element of group because G is abelian.

Ex :- 2 Consider the group G.

Then the Centre of Z of group G is a normal subgroup of G.

sol:- We recall the definition of the Centre of Z, we have

$Z = \{x \in G : xz = zx \ \forall x \in G\}$

We have Z is a subgroup of G.

Now, to show Z is normal in G.

For all $x \in G$ and $z \in Z$, we have

$$xzx^{-1} = zxx^{-1} \qquad (\because xz = zx)$$
$$= ze = z \in Z$$
$$\Rightarrow \quad xzx^{-1} \in Z$$

Hence Z is normal subgroup of G.

CONJUGATE ELEMENT :-

Let G be a group. An element $a \in G$ is called conjugate to an element $b \in G$ if and only if $a = x^{-1}bx$ for some $x \in G$.

If $a = x^{-1}bx$, then sometimes, we say that a is transform of b by x.

Notes :-

(i) Here, the element x is not unique.

(ii) If a is conjugate to b then we write $a - b$ and this relation is known as relation of conjugacy.

(iii) The relation of conjugacy is an equivalence relation.