

$$\left(\frac{\partial \mu_1}{\partial P}\right)_T dP + \left(\frac{\partial \mu_1}{\partial T}\right)_P dT = \left(\frac{\partial \mu_2}{\partial P}\right)_T dP + \left(\frac{\partial \mu_2}{\partial T}\right)_P dT$$

or

$$\left(\frac{\partial \mu_2}{\partial P} - \frac{\partial \mu_1}{\partial P}\right)_T dP = -\left(\frac{\partial \mu_2}{\partial T} - \frac{\partial \mu_1}{\partial T}\right)_P dT$$

or

$$\frac{dP}{dT} = -\frac{\left(\frac{\partial \mu_2}{\partial T} - \frac{\partial \mu_1}{\partial T}\right)_P}{\left(\frac{\partial \mu_2}{\partial P} - \frac{\partial \mu_1}{\partial P}\right)_T}$$

(2)

and from Maxwell's relations

$$\left(\frac{\partial \mu}{\partial T}\right)_{P,N} = -\left(\frac{\partial S}{\partial N}\right)_{P,T}$$

and

$$\left(\frac{\partial \mu}{\partial P}\right)_{T,N} = \left(\frac{\partial V}{\partial N}\right)_{P,T}$$

(3)

Putting this value in Eq. (2), we get

$$\frac{dP}{dT} = -\frac{\left(\frac{\partial S}{\partial N_2} - \frac{\partial S}{\partial N_1}\right)}{\left(\frac{\partial V}{\partial N_2} - \frac{\partial V}{\partial N_1}\right)}$$

(4)

Where all the derivatives are at constant P and T. For given P and T, S and V are linear function of the N_i . Then Eq.(4) is written as

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V}$$

(5)

Where ΔS and ΔV are, respectively, the change in entropy and volume for a given number of particles. Integrating Eq.(5) gives the coexistence curve, i.e. the relation between P and T which must hold if the two phases are in equilibrium.

The energy which must be supplied to change one K-mole (N particles) from phase 1 to phase 2 at constant pressure and temperature (i.e. the enthalpy change ΔH) is the K-mole latent heat L. At constant pressure, $dH = TdS$, so that $L = T\Delta S$ and Eq.(5) can be written as

$$\frac{dP}{dT} = \frac{L_{vap.}}{T \Delta V_m}$$

(6)

Where ΔV_m is the change in volume per K-mole. This relation, which gives the slope of the coexistence curve, is called Clausius-Clapeyron equation. Since L is positive by definition, the sign of ΔV_m determine the sign of the slope. The same reasoning applied to the solid-liquid and solid – vapour phases gives the corresponding Clausius-Clapeyron relations.