I^st order and I^st degree differential Equation

Separation of variables :-

Exp. Solve $\frac{dy}{dx}\sin x - y\cos x + y^2 = 0$

Solution :- Given diff. Ep.

$$\frac{dy}{dx}\sin x - y\cos x + y^2 = 0$$

$$\frac{1}{y^2}\sin x\frac{dy}{dx} - \frac{1}{y}\cos x + 1 = 0$$

$$\frac{d}{dx}\left(-\frac{1}{y}\right)\sin x + \left(-\frac{1}{y}\right)\frac{d}{dx}(\sin x) + 1 = 0$$

$$\frac{d}{dx}\left[-\frac{\sin x}{y}\right] + 1 = 0$$

Integrating both sides.

$$-\frac{\sin x}{y} + x = c$$

$$\sin x = y(x - c)$$

=

Que. 1 Solve $(x+y)^2\frac{dy}{dx} = a^2$ (Hint put $x+y=z$

Que. 2 Solve $x\,dx + y\,dy = \frac{a^2(x\,dy - y\,dx)}{x^2+y^2}$ Hint put $x = r\cos\theta$, $y = r\sin\theta$, then $x^2+y^2 = r^2$, $\tan\theta = \frac{y}{x}$

Que-3 Solve $\log_e\left(\frac{dy}{dx}\right) = ax + by$

Hint $\frac{dy}{dx} = e^{ax+by}$

Exp. Solve $\frac{dy}{dx} = \frac{y^2+y+1}{x^2+x+1}$

Solution Given differential Equation

$$\frac{dy}{dx} = \frac{y^2+y+1}{x^2+x+1}$$

$$\frac{dy}{y^2+y+1} = \frac{dx}{x^2+x+1}$$

$$\frac{dy}{(y+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = \frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

Integrating both sides, we get

$$\frac{2}{\sqrt{3}}\tan^{-1}\frac{y+\frac{1}{2}}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}\tan^{-1}\frac{x+\frac{1}{2}}{\sqrt{3}/2} + \frac{2}{\sqrt{3}}\tan^{-1}\frac{3c}{\sqrt{3}}$$ (Constant)

$$\tan^{-1}\frac{2y+1}{\sqrt{3}} - \tan^{-1}\frac{2x+1}{\sqrt{3}} = \tan^{-1}\frac{3c}{\sqrt{3}}$$

$$\tan^{-1}\frac{\frac{2y+1}{\sqrt{3}} - \frac{2x+1}{\sqrt{3}}}{1+\frac{(2y+1)}{\sqrt{3}}\cdot\frac{(2x+1)}{\sqrt{3}}} = \tan^{-1}\frac{3c}{\sqrt{3}}$$

$$\tan^{-1}\frac{(2y+1-2x-1)/\sqrt{3}}{\frac{(3+4xy+2x+2y+1)}{3}} = \tan^{-1}\frac{3c}{\sqrt{3}}$$

$$\frac{3(2y-2x)}{\sqrt{3}(3+4xy+2x+2y)} = \frac{3c}{\sqrt{3}}$$

$$\frac{(y-x)}{2+2xy+x+y} = c \Rightarrow (y-x) = c(x+y+2xy+2)$$