Linear differential Equation

Exp. Solve $\frac{dy}{dx} + \frac{1-2x}{x^2} \cdot y = 1$

Solution The given differential equation is of the form $\frac{dy}{dx} + Py = Q$

$$P = \frac{1-2x}{x^2}$$

$$Q = 1$$

Then

$$I.F = e^{\int P dx}$$

$$= e^{\int \frac{1-2x}{x^2} dx}$$

$$= e^{\int \frac{1}{x^2} dx - 2\int \frac{dx}{x}}$$

$$= e^{-\frac{1}{x}} \cdot e^{-2\log x}$$

$$= e^{-\frac{1}{x}} e^{\log x^{-2}} = e^{-\frac{1}{x}} \cdot x^{-2} =$$

$$I.F = e^{-\frac{1}{x}} \cdot \frac{1}{x^2}$$

$$y \cdot I.F = \int I.F \cdot Q \, dx$$

$$y \cdot e^{-\frac{1}{x}} \cdot \frac{1}{x^2} = \int 1 \cdot e^{-\frac{1}{x}} \frac{1}{x^2} dx$$

put $\frac{1}{x} = z$

$\therefore -\frac{1}{x^2} dx = dz$

$$y \cdot e^{-\frac{1}{x}} \cdot \frac{1}{x^2} = -\int e^{-z} dz$$

$$y \cdot e^{-\frac{1}{x}} \cdot \frac{1}{x^2} = e^{-z} + c$$

$$y \cdot e^{-\frac{1}{x}} \cdot \frac{1}{x^2} = e^{-\frac{1}{x}} + c$$

$$y = x^2 (1 + c e^{\frac{1}{x}})$$

Exp. Solve $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

Solution Given diff. Eqn is

$$(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$$

$$(1+y^2)\frac{dx}{dy} = \tan^{-1}y - x$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{1}{1+y^2}\cdot x = \frac{\tan^{-1}y}{1+y^2}$$

This Equation is of the form

$$\frac{dx}{dy} + Px = Q$$

therefore

$$I.F = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

$$x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} \cdot e^{\tan^{-1}y}\,dy + c$$

$$= \int e^{\tan^{-1}y} \tan^{-1}y \frac{dy}{1+y^2} + c$$

Putting $\tan^{-1}y = z$

$$\frac{1}{1+y^2}dy = dz$$

Then

$$x \cdot e^{\tan^{-1} y} = \int \underset{II}{e^z} \underset{I}{z}\, dz + c$$

$$= z \cdot e^z - \int 1 \cdot e^z dz + c$$

$$= z \cdot e^z - e^z + c$$

$$x \cdot e^{\tan^{-1} y} = e^z (z-1) + c$$

$$x \cdot e^{\tan^{-1} y} = e^{\tan^{-1} y} (\tan^{-1} y - 1) + c$$

$$x = (\tan^{-1} y - 1) + \frac{c}{e^{\tan^{-1} y}}$$

$=$