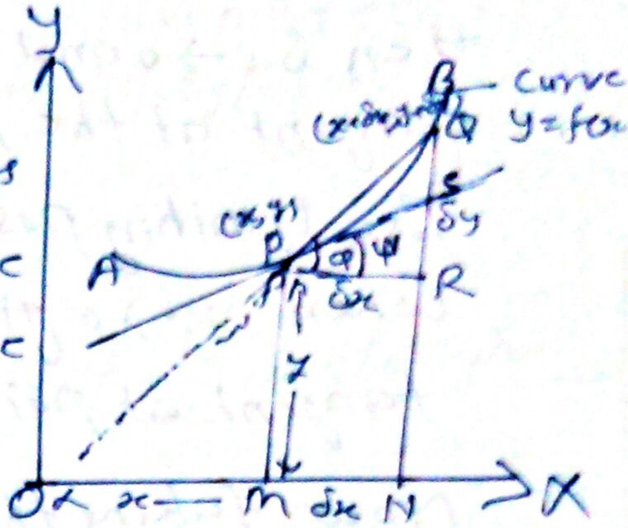
Tangent → Let $P$ and $Q$ be any two points on a given curve ($y=f(x)$).

When the point $Q$ moves along the curve, concides with the point, when the limiting position of the secant $PQ$ is called the tangent at the point $P$. That is, a tangent is a straight line passing through two coincident points $P$ and $Q$.




Let the Equation of the curve be

$$y=f(x)$$

Let $P(x, y)$ and $Q(x+\delta x,\ y+\delta y)$ be two points on this curve.

$\quad\ \ x_1,\ y_1 \qquad\qquad\qquad x_2 \qquad\quad y_2$

The Equation of the line $PQ$ is
Let $(X,Y)$ denote the current co-ordinate

$$Y-y_1=\frac{y_2-y_1}{x_2-x_1}\cdot(X-x_1)$$

$$Y-y=\frac{y+\delta y-y}{x+\delta x-x}\,(X-x)$$

$$Y-y=\frac{\delta y}{\delta x}(X-x) \qquad \text{—— (1)}$$

From figure $PM$ and $QN$ are perpendiculars on the axis $X$. Then

$OM=x\ ,\ ON=x+\delta x \Rightarrow MN=\delta x$

$PM=y\ ,\ QN=y+\delta y \Rightarrow QR=\delta y$

Now we move the point Q along the curve to meet with the point P i.e. Q→P then $\delta x \to 0$ and also chord PQ becomes the tangent at the point P.

The limiting position of the equation (1) when $\delta x \to 0$ gives the equation of the tangent at point $P(x, y)$.

Now taking the limit of Eqn (1) as $\delta x \to 0$ we get

$$Y - y = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}(X - x)$$

$$Y - y = \frac{dy}{dx}(X - x)$$

This is the Eqn of the tangent at the point $P(x, y)$.

$\Rightarrow \frac{dy}{dx}$ is the slope of the tangent

$\therefore \frac{dy}{dx} = \tan\varphi$ $\qquad \angle SPR = \varphi$

$\angle QPR = \theta$

Thus if the tangent is parallel to x-axis then $\frac{dy}{dx} = 0$

* If we take $(x, y)$ as current co-ordinates then the Eqn of the tangent at point $(x_1, y_1)$ as

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}(x - x_1)$$

$\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ i.e. put $x = x_1$ & $y = y_1$ in the diff. coeff.

Exp. Prove that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-x/a}$ at the point where the curve crosses the axis of y.

Sol:- The x-co-ordinate of the curve cuts at the point the y axis is 0.

In order to find the y co-ordinate of the point putting $x=0$ in the given curve

$$y=be^{-x/a} = be^{0}=b$$

point $(0, b)$

Given the curve $y=be^{-x/a}$

Diff. w.r.t x.

$$\frac{dy}{dx}=b.\,e^{-x/a}\left(-\frac{1}{a}\right) = \frac{-b}{a}e^{-x/a}$$

$$\left(\frac{dy}{dx}\right)_{\substack{x=0\\y=b}} = \frac{-b}{a}.\,e^{0} = \frac{-b}{a}$$

Hence the required Equation of the tangent at the point $(0,b)$ is

$$(y-b)=\frac{-b}{a}(x-0)$$

$$y-b=\frac{-b}{a}x$$

$$ay-ab=-bx \Rightarrow bx+ay=ab$$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}=1$$