B.Sc. I.
P-2. Group-B.

Discuss Debye theory of specific heat of Solid.

Debye's theory of sp. heat is based on more logical grounds than that of Einstein. Atoms in a solid are very close to each other their vibrations cannot be considered independent. Thus, Debye considered the vibrational modes of a crystal as a whole and not of individual atoms as was done by Einstein, assuming the atomic vibrations to be independent of each other.

Debye considered that, the vibrations would form a continuous spectrum and these vibrations are identical with the elastic vibrations of a continuous solid.

When a continuous solid is subjected to elastic vibrations, two kinds of vibrations are produced: (i) transverse vibrations, which travel with the velocity $C_t = (\eta/\rho)^{1/2}$ and (ii) Longitudinal vibrations which travel with the velocity $C_l = \left[(K + \frac{4}{3}\eta)/\rho\right]^{1/2}$

where η = coefficient of rigidity and
K = bulk modulus.

The no. of modes of longitudinal vibrations per unit volume having frequencies ν and $\nu + d\nu$ is

$$= \frac{4\pi \nu^2 d\nu}{C_l^3} \quad \text{--- (1)}$$

and the no. of modes of transverse vibrations per unit volume between the same limits:

$$= 2 \cdot \frac{4\pi V \nu^2 d\nu}{C_t^3} \quad \text{———} \quad (2)$$

The total number of independent modes of vibration is now given by.

$$\int_0^{\nu_m} 4\pi V \left[\frac{1}{C_l^3} + \frac{2}{C_t^3}\right] \nu^2 d\nu \quad \text{———} \quad (3)$$

$$= 4\pi V \left(\frac{1}{C_l^3} + \frac{2}{C_t^3}\right) \int_0^{\nu_m} \nu^2 d\nu$$

The upper limit of integration cannot be infinite, as this would render the number of degrees of freedom infinite. Actually this number is 3N, where N is the number of particles in volume V.

Thus

$$\int_0^{\nu_m} 4\pi V \left[\frac{1}{C_l^3} + \frac{2}{C_t^3}\right] \nu^2 d\nu = 3N$$

Since N atoms are equal to 3N vibrations in mutually three perpendicular arrays:

or, $$4\pi V \left[\frac{1}{C_l^3} + \frac{2}{C_t^3}\right] \frac{\nu_m^3}{3} = 3N$$

or, $$4\pi V \left[\frac{1}{C_l^3} + \frac{2}{C_t^3}\right] = \frac{9N}{\nu_m^3}$$

or, $$\nu_m^3 = \frac{9N}{4\pi V \left[\frac{1}{C_l^3} + \frac{2}{C_t^3}\right]} \quad \text{———} \quad (4)$$

The total thermal energy for 1 gm-atom of solid in the frequency range 0 and ν_m.

$$U = \frac{9N}{\nu_m^3} \int_0^{\nu_m} \frac{h\nu^3 \, d\nu}{e^{h\nu/KT} - 1} \quad \text{—— (5)}$$

Let $\frac{h\nu}{KT} = \xi$; $\frac{h\nu_m}{k} = \theta$

$$\therefore d\xi = \frac{h}{KT} d\nu \; ; \; \frac{\theta}{T} = \xi_m = x \quad \text{—— (6)}$$

using eqn. (6) in eqn. (5), we get for U

$$U = 3RT \cdot 3\left(\frac{T}{\theta}\right)^3 \int_0^{\xi_m} \frac{\xi^3 \, d\xi}{e^{\xi} - 1} \quad \text{—— (7)}$$

Where, $Nk = R$.

Eqn. (7) cannot be integrated in finite terms. Now,

$$C_v = \frac{\delta U}{\delta T} = 3R\,D(x) \quad \text{—— (8)}$$

Where, $D(x) = \left[\frac{12}{x^3} \int_0^{x} \frac{\xi^3 \, d\xi}{e^{\xi} - 1} - \frac{3x}{e^{x} - 1}\right]$.

being a function of x only.

The function $D(x)$ is known as the Debye functions and θ is known as the Debye temperature of the solid $(\theta = h\nu_m / k)$.

Case I. At $T >> \theta$, x and x_D are very small.

$$C_v = 3R = 3Nk \quad \text{——— (9)}$$

Case II. At $T << \theta$, x and x_D are large.

$$C_v = \frac{12\pi^4}{5} R\left(\frac{T^3}{\theta}\right) = 234\, R\left(\frac{T}{\theta}\right)^3 \quad \text{— (10)}$$

or, $C_v \propto T^3$. ——— (11)

This is known as Debye's T^3 law.

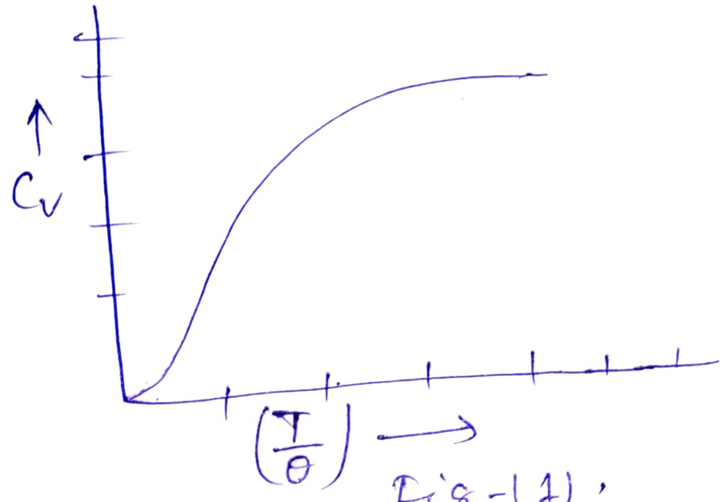

Fig-(1).

Heat capacity C_v of a solid, according to Debye approximation is shown in fig. (1).