were made to develop relativistic quantum theory.

15·1. KLEIN GORDAN EQUATION

The relativistic relation between total energy and momentum of a particle is given by

$$E = \sqrt{p^2 C^2 + m^2 C^4} \quad \text{...(15·1)}$$

So one can expect, Hamiltonian of a free particle in accordance with $H\psi = E\psi$

$$H = \sqrt{p^2 C^2 + m^2 C^4}$$

Transition from classical theory to quantum theory is achieved by treating p and H as operators. In coordinate representation, we have

$$\vec{p} \equiv -i\hbar \nabla$$

HΨ = EΨ

$$\vec{E} \equiv i\hbar \frac{\partial}{\partial t} = H$$

Relativistic quantum equation of motion will now read as—

$$i\hbar \frac{\partial \psi}{\partial t} = \left(\sqrt{-\hbar^2 C^2 \nabla^2 + m^2 C^4}\right) \psi \quad \text{...(15·2)}$$

Right-hand side of the above equation contains under-root of an operator. We can get rid of square root of an operator by expanding it in terms of a power series. However, this expansion introduces a few difficulties :

(1) Equation (15·2) after expansion will have space and time derivatives in unsymmetrical form and thereby a covariant theory cannot be formulated.

(2) Right-hand side will contain a large number of ~~atoms~~ terms involving even powers of ∇ ; making it difficult to be solved.

These difficulties are removed, if we formulate the equation with …uare of Hamiltonian instead of H.

$$H^2 = p^2 C^2 + m^2 C^4$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -\hbar^2 C^2 \nabla^2 \psi + m^2 C^4 \psi \qquad \text{...(15·3)}$$

…nd (This is possible for a commuting set of operators A and B,

…f $A\psi = B\psi$

…hen $A^2\psi = A\, A\psi = AB\psi = BA\psi = BB\psi = B^2\psi$.

$$\therefore \quad \left(\hbar^2 C^2 \nabla^2 - \hbar^2 \frac{\partial^2}{\partial t^2} - m^2 C^4 \right) \psi = 0$$

or

$$\left[\nabla^2 - \frac{1}{C^2} \frac{\partial_2}{\partial t^2} - \frac{m^2 C^2}{\hbar^2} \right] \psi = 0$$

or

$$\left[\Box^2 - \frac{m^2 C^2}{\hbar^2} \right] \psi = 0 \qquad \text{...(15·4)}$$

Where $\Box^2 = \nabla^2 - \frac{1}{C^2} \frac{\partial^2}{\partial t^2} = \frac{\partial}{\partial x_\mu} \cdot \frac{\partial}{\partial x_\mu}$ and is known as *de-Alemberation operator*.

Equation (15·4) was proposed by ***Klein and Golden*** and is known as ***Klein-Golden equation*** or ***Schrodinger's relativistic equation*** in literature.

Equation (15·4) has plane wave solutions of the form

$$\psi = \exp.\, (i \vec{K} . \vec{r} - i\omega t) \qquad \text{...(15·5)}$$

These are eigen functions of energy and momentum with eigen values $\hbar\omega$ and $\hbar\vec{K}$ respectively, It is apparent that (15·5) satisfies (15·4)

if

$$-K^2 + \frac{\omega^2}{C^2} - \frac{m^2 C^2}{\hbar^2} = 0$$

or

$$\hbar^2 \omega^2 = \hbar^2 K^2 C^2 + m^2 C^4$$

or

$$\hbar\omega = \pm\sqrt{\hbar^2 K^2 C^2 + m^2 C^4} \qquad \text{...(15·6)}$$

The positive and negative square roots in (15·6) correspond to an ambiguity in the sign of energy. However, we shall postpone the discussion of negative energy solution for the time being ; as it will be interpreted to be antiparticle solution.