Definition:

Let H be a subgroup of the group G whose operation is written multiplicatively. Given an element g of G, the left cosets of H in G are the sets obtained by multiplying each element of H by a a fixed element g of G (where g is the left factor). In symbols these are,

$$gH = \{gh : h \text{ an element of } H\} \text{ for each } g \text{ in } G.$$

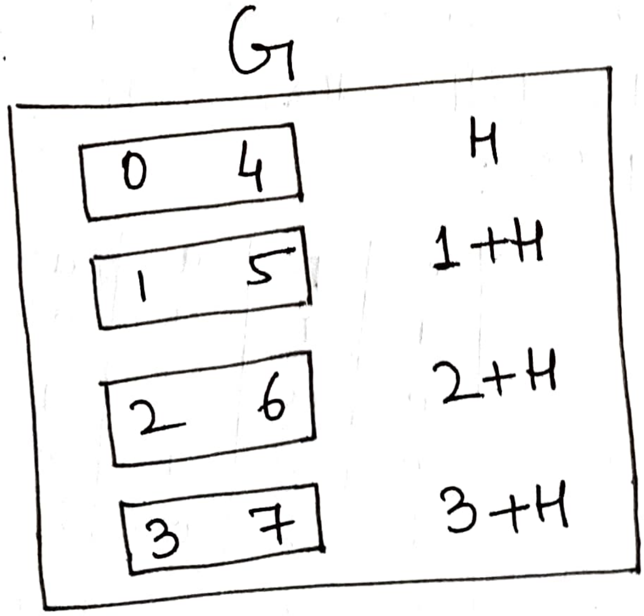
The right cosets are defined similarly, except that the element g is now a right factor, that is,

$Hg = \{hg : h \text{ an element of } H\}$ for g in G.

As g varies through the group, it would appear that many cosets (right or left) would be generated. This is true, but two left cosets (respectively right cosets) are either distinct or are identical as sets.

If the group operation is written additively, as is often the case when the group is abelian, the notation used changes to $g+H$ or $H+g$, respectively.

<u>Example</u>



G is the group $(\mathbb{Z}/8\mathbb{Z}, +)$, the integers mod 8 under addition. The subgroup

4)

H contains only 0 and 4. there are four left cosets of H: H itself, 1+H, 2+H, and 3+H (written using additive notation since this is the additive group). Together they partition the entire group G into equal-size, non overlaping sets. The index $[G:H]$ is 4

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First example!.

Let G be the dihedral group of order six. Its elements may be represented by $\{I, a, a^2, b, ab, a^2b\}$. In this group, $a^3 = b^2 = I$ and $ba = a^{-1}b$. This is enough information to fill in the entire multiplication table:

*	I	a	a^2	b	ab	a^2b
I	I	a	a^2	b	ab	a^2b
a	a	a^2	I	ab	a^2b	b
a^2	a^2	I	a	a^2b	b	ab
b	b	a^2b	ab	I	a^2	a
ab	ab	b	a^2b	a	I	a^2
a^2b	a^2b	ab	b	a^2	a	I

Let T be the subgroup $\{I, b\}$. The (distinct) left cosets of T are:

$$IT = T = \{I, b\},$$
$$aT = \{a, ab\}, \text{ and}$$
$$a^2T = \{a^2, a^2b\}.$$

Since all the elements of G have now appeared in one of these cosets, generating any more can not give new cosets, since a new cosets would have to have an element in common with one of these and therefore be identical to one of these cosets. For instance,

$$abT = \{ab, a\} = aT.$$

The right cosets of T are:

$$TI = T = \{I, b\}$$
$$Ta = \{a, ba\} = \{a, a^2b\}, \text{ and}$$
$$Ta^2 = \{a^2, ba^2\} = \{a^2, ab\}.$$

For this example, except for T, no left coset is also a right coset.

Let H be the subgroup $\{I, a, a^2\}$. The left cosets of H are $IH = H$ and $bH = \{b, ba, ba^2\}$.

The right cosets of H are $HI = H$ and $Hb = \{b, ab, a^2b\} = \{b, ba^2, ba\}$. In this case, every left coset of H is also a right coset of H.

Let H be a subgroup of a group G and suppose that $g_1, g_2 \in G$. The following statement are equivalent:

* $g_1 H = g_2 H$
* $Hg_1^{-1} = Hg_2^{-1}$
* $g_1 H \subset g_2 H$
* $g_2 \in g_1 H$
* $g_1^{-1} g_2 \in H$.