

SU(2) symmetry

The process of forming different iso-spin multiplets from a number of isospin doublets is carried out by the rules of a special type of transformation known as the special unitary group of rank 2 (i.e. SU(2)) the number 2 in the bracket indicates dimensions of group. It is group of all 2×2 unitary matrices, where determinant.

It is well known that neutron and proton are two states of nucleon which form the fundamental representation of the group. The symmetry operations can transfer a proton into neutron or neutron into proton. This implies that iso-spin will remain conserved under strong interaction :-

The neutron and proton each has iso-spin $T = \frac{1}{2}$.

The third component $T_3 = \frac{1}{2}$ for proton.

$T_3 = -\frac{1}{2}$ for neutron.

The operators of symmetry group thus change the co-ordinates of iso-spin is such away as to reverse the sign of T_3 . Thus the strong interactions are assumed to be invariant under rotations in the iso-spin space.

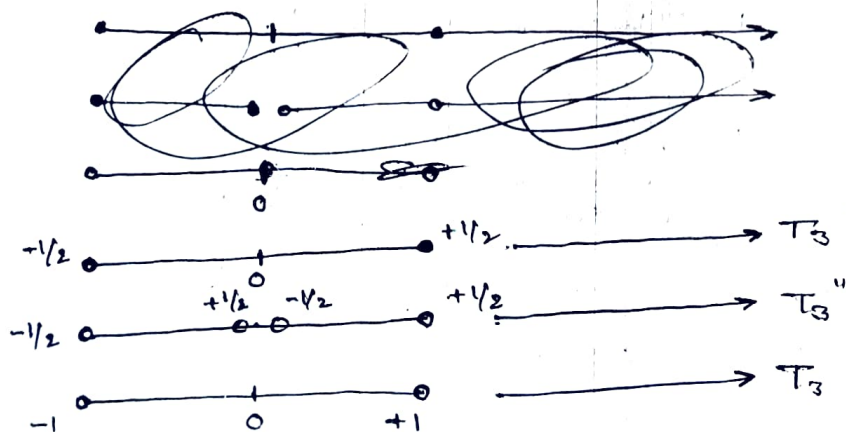


Diagram shows the generating of iso-spin triplet

and an isospin singlet from two iso-spin doublet.

From group theory, we ~~have~~ ^{know} that two isospin $\frac{1}{2}$ states are the simplest basic states known as $SU(2)$ group of transformations. The Pauli-spin matrices are the simplest representation of this group which is intimately related to the group of rotation in iso-spin space. which leads to the conservation of angular momentum and iso-spin.

The unitary group is special because a restriction reduces by unity to the number of operators in the group.

Thus in $SU(2)$ group, instead of $2 \times 2 = 4$ operators there are three operators. The group is said to have generator.

In symbolically

$$2 \otimes \bar{2} = 3 \oplus 1.$$

2 & $\bar{2}$ signify doublet ($T = \frac{1}{2}$) isospin states for a ~~nucleon~~ nucleon and antinucleon respectively.

3 and 1 signify the resultant triplet ($T = 1$) and singlet ($T = 0$) iso-spin states resulting from the combination of two such doublets. The iso-spin T determines the multiplicity $(2T+1)$ whose third components varies from $T_3 = +T$ to $T_3 = -1$. These substates are identical except for the electric charge. eg. for pions. —

$$\text{iso spin } T = 1$$

$$\text{multiplicity } (2T+1) = 3$$

There are three charge states of pions with

$$T_3 = +1 \text{ for } \pi^+ \rightarrow p \bar{n}$$

$$T_3 = 0 \text{ for } \pi^0 \rightarrow \frac{1}{\sqrt{2}} (n \bar{n} - p \bar{p})$$

$$T_3 = -1 \text{ for } \pi^- \rightarrow (-) n \bar{p}$$

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The iso spin is strictly conserved in strong interactions, the sub-states of the multiplets would differ in charge and T_3 but not in mass. The ~~is~~ $SU(2)$ symmetry is violated in electromagnetic interactions and also ~~breaks~~ breaks in the case of weak interactions.