⇒ De-Morgan's theorem:-

Theorem I:- The complement of the sum of two or more variables is equal to the product of the complements of the variables.

If A & B are two variables.

$$\overline{A+B} = \overline{A}\cdot\overline{B} \quad \text{——— (1)}$$

Theorem II:- The complement of the product of two or more variables is equal to the sum of the complements of the variables.

$$\overline{A\cdot B} = \overline{A} + \overline{B} \quad \text{——— (2)}$$

→ To prove $\overline{A+B} = \overline{A}\cdot\overline{B}$

(i) when $A = 0$, $B = 0$.

$\overline{A+B} = \overline{0+0} = \overline{0} = 1$

and $\overline{A}\cdot\overline{B} = \overline{0}\cdot\overline{0} = 1\cdot 1 = 1$

Hence. $\overline{A+B} = \overline{A}\cdot\overline{B}$.

(ii) when $A=0$, $B=1$, $\overline{A+B} = \overline{0+1} = \overline{1} = 0$

and $\overline{A}\cdot\overline{B} = \overline{0}\cdot\overline{1} = 1\cdot 0 = 0$

Hence. $\overline{A+B} = \overline{A}\cdot\overline{B}$.

(iii) when $A=1$, $B=0$, $\overline{A+B} = \overline{1+0} = \overline{1} = 0$.

and $\overline{A}\cdot\overline{B} = \overline{1}\cdot\overline{0} = 0\cdot 1 = 0$.

Hence, $\overline{A+B} = \overline{A}\cdot\overline{B}$

(iv) when $A=1$, $B=1$, $\overline{A+B} = \overline{1+1} = \overline{1} = 0$.

and $\overline{A}\cdot\overline{B} = \overline{1}\cdot\overline{1} = 0\cdot 0 = 0$ Hence. $\overline{A+B} = \overline{A}\cdot\overline{B}$.

→ To prove $\overline{A \cdot B} = \overline{A} + \overline{B}$

(i) When $A=0$, $B=0$. $\overline{A \cdot B} = \overline{0 \cdot 0} = \overline{0} = 1$

and $\overline{A} + \overline{B} = \overline{0} + \overline{0} = 1 + 1 = 1$

Hence, $\overline{A \cdot B} = \overline{A} + \overline{B}$.

(ii) When, $A = 0$, $B = 1$.

$\overline{A \cdot B} = \overline{0 \cdot 1} = \overline{0} = 1$

and $\overline{A} + \overline{B} = \overline{0} + \overline{1} = 1 + 0 = 1$

Hence. $\overline{A \cdot B} = \overline{A} + \overline{B}$.

(iii) When. $A = 1$, $B = 0$. $\overline{A \cdot B} = \overline{1 \cdot 0} = \overline{0} = 1$

and $\overline{A} + \overline{B} = \overline{1} + \overline{0} = 0 + 1 = 1$

Hence. $\overline{A} \cdot \overline{B} = \overline{A} + \overline{B}$

(iv) When. $A = 1$ & $B = 1$

$\overline{A \cdot B} = \overline{1 \cdot 1} = \overline{1} = 0$

and $\overline{A} + \overline{B} = \overline{1} + \overline{1} = 0 + 0 = 0$.

Hence, $\overline{A \cdot B} = \overline{A} + \overline{B}$.

Thus De-morgon's theorem proved.