Solution of a differential Equation:-

A solution (or integral) of a diff. equation is a relation between the variables which satisfies the given differential equation.

Exp. $x = A\cos(nt+\alpha)$ is a solution of diff. Eqn. — (1)

$$\frac{d^2x}{dt^2} + n^2x = 0. \quad \text{— (2)}$$

→ The general (or complete) solution of a differential equation is that in which the number of arbitrary constants is equal to the order of the diff. Eqn.

Thus, the number of arbitrary constants (A, α) is the same as the order of diff. Eqn. (2).

→ A particular solution is that which can be obtained from the general solution by giving particular values to the arbitrary constants.

Exp. $x = A\cos(nt + \pi/4)$ is the particular solution of the Eqn (2) when putting $\alpha = \pi/4$ in equation (1).

* Differential Equations of the first order and first degree:

$$\frac{dy}{dx} + P(x)y = \phi(x),$$

where P & Q are function of x. This equation is called first order and first degree because it only involves the function y and its derivative i.e. dy/dx.

* First order and first degree differential equations:- It is not possible to solve such equations in general. Some special methods of solution, which are applicable to the following types of equations-

(i) Equations where variables are separable
(ii) Homogenous equations
(iii) Linear equations
(iv) Exact equations.

(1)(a) Variables Separable:- In the diff. Equ., to collect all functions of x and dx on one side and all the functions of y and dy on the other side, then the variables are said to be separable. Thus the general form of such an equation is

$$f(y)\,dy = \phi(x)\,dx$$

Then integrating both sides we get,

$$\int f(y)\,dy = \int \phi(x)\,dx + c$$

as final solution of this

by variables separable

Exp. 1 $x\,dy = (2x^2+1)\,dx$

$$dy = \frac{(2x^2+1)}{x}\,dx$$

$$dy = \left(2x + \frac{1}{x}\right)dx$$

$$dy = 2x\,dx + \frac{1}{x}\,dx$$

Integrating both side

$$\int dy = 2\int x\,dx + \int \frac{1}{x}\,dx + C$$

$$y = 2\frac{x^2}{2} + \log|x| + C$$

$$y = x^2 + \log|x| + C$$

Exp. 2 Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$

Given Eqn. $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$

$$\frac{dy}{dx} = e^{-2y}\left(e^{3x} + x^2\right)$$

$$\frac{dy}{e^{-2y}} = e^{3x}\,dx + x^2\,dx$$

Integrating $\int e^{2y}\,dy = \int e^{3x}\,dx + \int x^2\,dx + C$

$$\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + C$$

$$3e^{2y} = 2\left(e^{3x} + x^3\right) + C' \quad \Rightarrow C' = 6C$$