Exp2- Form the differential equation of curves $y = a\sin(x+b)$, where a, b are arbitrary constants.

Sol. Similar solution as Exp. 1.

* The numbers of constants in curve $y = a\sin(x+b)$ are two, therefore numbers of constants is equal to the number of time we differentiate.

$$y = a\sin(x+b) \quad \text{---} (1)$$

Diff. w. r. to x

$$\frac{dy}{dx} = \frac{d(a\sin x+b)}{dx}$$

$$\frac{dy}{dx} = a\cos(x+b)\cdot 1$$

Again differentiating

$$\frac{d^2y}{dx^2} = -a\sin(x+b)$$

$$\frac{d^2y}{dx^2} = -y \quad \text{from (1)}$$

$$\frac{d^2y}{dx^2} + y = 0 \Rightarrow y = -\frac{d^2y}{dx^2}$$

So, this curve is form of diff. Eqn. as

Exp. Form the differential Equation of all circles of radius a.

Solution. Such a general equation of circle is

$(x-h)^2 + (y-k)^2 = a^2$, where h, k are the Co-ordinates of the circle with respect to x, y axises and radius a is constants.

Here h, k two arbitrary constants of variables, So it differenti-ate twice.

$$(x-h)^2 + (y-k)^2 = a^2 \quad \text{--- (i)}$$

Differentiate w.r.to x

$$2(x-h) + 2(y-k)\cdot\frac{dy}{dx} = 0$$

$$(x-h) + (y-k)\frac{dy}{dx} = 0 \quad \text{--- (ii)}$$

Again differentiate w.r.to x

$$1 + (y-k)\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{dy}{dx} - 0\right) = 0$$

$$1 + (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$(y-k) = -\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$

and $(x-h) = -(y-k)\frac{dy}{dx}$ from (ii)

We get from (i)

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$$

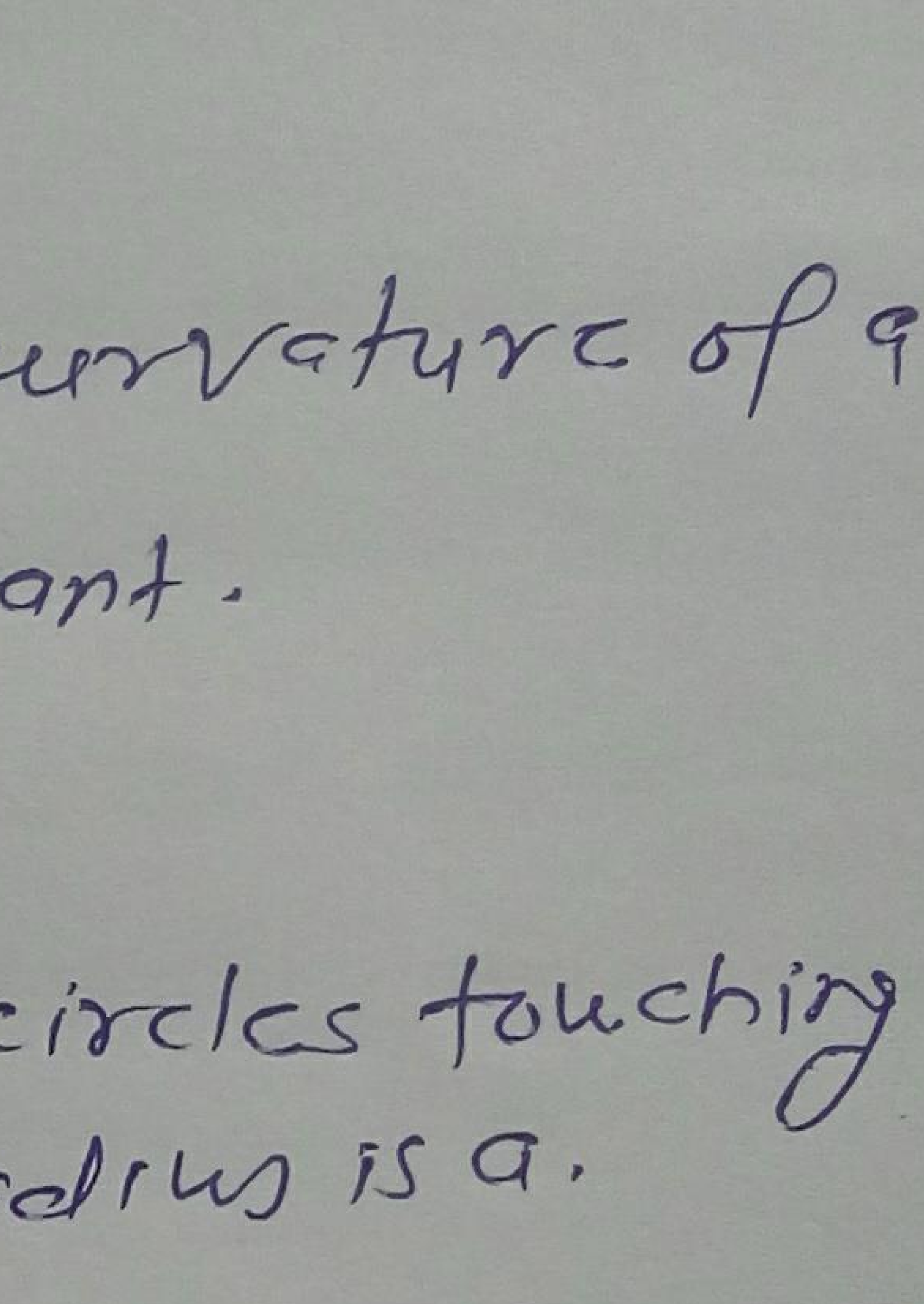
$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = a\frac{d^2y}{dx^2}$$ → This is in diff. Equ. form

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = a$$

It states that the radius of curvature of a circle at any point is constant.

Exp. Form of diff. Eqn. of the circles touching the x-axis at origin and radius is a.

Sol. Equation of circle $(x-h)^2+(y-k)^2 = a^2$
centre (h, k) & radius is a.



When circles touching the x-axis at origin then centre will be on y-axis (i.e $h=0$) then radius $a=k$

Thus, Equation of circle is

$$(x-0)^2+(y-k)^2 = k^2$$

$$x^2+(y-k)^2 = k^2$$

$$x^2+y^2+k^2-2ky = k^2$$

$$x^2+y^2 = 2ky \quad \text{——— (1)}$$

Since there is one arbitrary constant k so it differentiate one time and can be solve.