Differential Equation

→ **Definition:-** A differential equation is an equation which involves differential coefficients or derivatives. Exp. (i) $e^{x}dx + e^{y}dy = 0$

(ii) $\frac{dy}{dx} = x \cdot e^{y-x^2}$

(iii) $\frac{d^2x}{dt^2} + n^2x = 0$

(iv) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$

all are differential equations.

→ An ordinary differential equation is that in which all the differential coefficients have reference to a single independent variable.

or; In which unknown function depends on single independent variables i.e $\frac{dy}{dx} = f(x)$ or $y' = f(x)$.

Equations (i) to (iii) are ODE.

→ A partial differential equation is that in which there are two or more independent variables and partially differential coefficients with respect to any of them. Eqn (4) is PDE.

or In which unknown functions depend on several independent variables i.e

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

→ Order of differential Equation is the order of the highest derivative in the equation.

Exp. $\frac{dy}{dx}$ → first order of derivative

$\frac{d^2y}{dx^2}$ → second ——————

$\frac{d^3y}{dx^3}$ → Third order of dervative

$\frac{d^ny}{dx^n}$ → nth order of derivative

→ The degree (power) of differential equation is the power of the highest derivative in the equation. Exp.

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = c\,\frac{d^2y}{dx^2}$$

$$\text{or } \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = c^2\left(\frac{d^2y}{dx^2}\right)^2$$

Power?
~~degree~~
order

$$c^2\left(\frac{d^2y}{dx^2}\right)^2 - \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = 0$$

In this equation, the highest derivative is 2, so this is second order and second degree differential equation.

* Differential Equations arise from many physical problems in oscillations of mechanical and electrical systems, bending of beams, conduction of heat, velocity of chemical reactions.

The approach of student to the study of differential equations, they are only intersted in solving the differential Eqns. without knowing as to how the differential equations are formed and how their solutions are physically interpreted.

→ Thus the study of differential equation consists of three phases.

(i) Formation of differential equation from the given problem

(ii) Solution of this diff. Eqn., evaluating the constants from the given conditions.

(iii) Physical interpretation of the solution.

In applied mathematics, every geometrical or physical problem when translated into mathematical symbols gives rise to differential equation.

* Formation of differential Equation:- An ordinary differential equation is formed in an attempt to eliminate certain arbitrary constant from a relation in the variables and constants.

Exp 1- Form the differential equation of simple harmonic motion given by

$$x = A\cos(nt+\alpha) \quad \text{---} \quad (1)$$

Sol. To eliminate the constants A and α from Eqn (1) by differentiating it twice, we have

Eqn. (1) differentrating w.r.to t.

$$\frac{dx}{dt} = -nA\sin(nt+\alpha).1$$

Again differentiating

$$\frac{d^2x}{dt^2} = -n^2A\cos(nt+\alpha)$$

Then $\frac{d^2x}{dt^2} = -n^2x$ from (1)

Acceleration

Thus $\frac{d^2x}{dt^2} + n^2x = 0$

Distance

This is differential equation which states that the acceleration varies as the distance (x) from the origin.