

Problem:-

① Calculate the force constant for HCl molecule given that stretching frequency $\nu = 2890 \text{ cm}^{-1}$.

Soln:

$$\text{we know that } \nu = \frac{1}{2\pi c} \sqrt{\frac{K}{\mu}} \text{ cm}^{-1}$$

on squaring both side we get -

$$\nu^2 = \frac{1}{4\pi^2 c^2} \cdot \frac{K}{\mu}$$

$$K = 4\pi^2 c^2 \mu \nu^2 \quad \text{--- (1)}$$

$$\mu_{\text{HCl}} = \frac{m_1 m_2}{m_1 + m_2} \quad [m_1 = 1 \quad m_2 = 35.5]$$

$$= \frac{1 \times 35.5}{1 + 35.5} = \frac{35.5}{36.5} \text{ amu}$$

$$= 0.972 \text{ amu}$$

$$\mu_{\text{HCl}} = 0.972 \times 1.66 \times 10^{-27} \text{ kg}$$

$$\mu_{\text{HCl}} = 1.614 \times 10^{-27} \text{ kg}$$

given,

$$\nu = 2890 \text{ cm}^{-1} = 2890 \times 10^2 \text{ m}^{-1}$$

$$\text{Thus, } K = 4 \times (3.14)^2 \times (2890 \times 10^2 \text{ m}^{-1})^2 \times (3 \times 10^8 \text{ m/sec})^2 \times 1.614 \times 10^{-27} \text{ kg}$$

$$K = 478.476 \text{ kg sec}^{-2}$$

$$K = 4.78 \times 10^2 \text{ kg sec}^{-2}$$

$$K = 4.78 \times 10^2 \text{ N m}^{-1}$$

$$[\because 1 \text{ N} = \text{kg m sec}^{-2}]$$

② The spectrum of HCl shows a very intense absorption at 2886 cm^{-1} , a weak one at 5668 cm^{-1} and a very weak one at 8347 cm^{-1} . Find out the equilibrium frequency (ω_e) of the molecule and anharmonicity constant (x_e) from these data —

Sol. Since, $\omega_e (1 - 2x_e) = 2886 \text{ cm}^{-1}$ — ①
 $2\omega_e (1 - 3x_e) = 5668 \text{ cm}^{-1}$ — ②
 $3\omega_e (1 - 4x_e) = 8347 \text{ cm}^{-1}$ — ③

on multiplying eqⁿ - ② by two (2) and then eqⁿ is ③ is subtracted from eqⁿ - ② we get —

$$2 \times [2\omega_e (1 - 3x_e) = 5668 \text{ cm}^{-1}]$$

~~$$4\omega_e (2 - 6x_e) = 11336 \text{ cm}^{-2}$$~~

~~$$\begin{array}{r} 4\omega_e - 12\omega_e x_e = 11336 \text{ cm}^{-2} \\ - 3\omega_e - 12\omega_e x_e = 8347 \text{ cm}^{-1} \\ \hline \end{array}$$~~

$$\boxed{\omega_e = 2989 \text{ cm}^{-1}}$$

$$\boxed{\text{i.e. } \omega_e = 2990 \text{ cm}^{-1} \text{ (approx.)}}$$

now putting the value of ω_e in eqⁿ - ② we get —

$$2 \times \omega_e (1 - 3x_e) = 5668 \text{ cm}^{-1}$$

~~$$2 \times 2990 \text{ cm}^{-1} (1 - 3x_e) = 5668 \text{ cm}^{-1}$$~~

~~$$1 - 3x_e = \frac{5668}{2 \times 2990} = \frac{2834}{2990}$$~~

$$1 - 3x_e = \frac{2834}{2990}$$

$$3x_e = 1 - \frac{2834}{2990}$$

$$3x_e = \frac{2990 - 2834}{2990}$$

$$x_e = \frac{156}{2990 \times 3} = \frac{26}{1495}$$

$$x_e = 0.0173$$

So, the equilibrium frequency (ω_e) and anharmonicity constant (x_e) are 2990 cm^{-1} and 0.0173 respectively.

Q Force constant $K = 4\pi^2 \bar{\omega}^2 c^2 \mu$

$K = 516 \text{ N m}^{-1}$ Ans.
 (1N = kg m s^{-2})

Q (3). Carbon monoxide shows an absorption band at 2140 cm^{-1} . Calculate the force constant of the molecule.

(Ans $K = 1840 \text{ N m}^{-1}$)

Q (4). From the fundamental vibrational frequency of CO at 2140 cm^{-1} . Calculate the frequency in Hz (sec^{-1}) and the zero point energy in kJ mole^{-1} .

Q (5). The rotation-vibration spectrum of HCl gas shows a number of lines ~~centered~~ centered about the fundamental vibrational band a few of which have the wave-numbers $2847, 2866, 2906$ and 2927 cm^{-1} . Using this data calculate —

(a) the bond length

(b) the force constant

(c) the anharmonicity constant of HCl.

Soln $\Delta E_{J,v} = \bar{\omega}_0 + 2Bm$
 $= \bar{\omega}_0 + \left(\frac{h}{4\pi^2 \mu c} \right) m^2$

($m = \pm 1, \pm 2, \pm 3$)

The pair of lines 2847 cm^{-1} and 2866 cm^{-1} (low wave no. side) belong to P-branch & pair of lines 2906 & 2927 cm^{-1} belong to R-branch (i.e. m is $+m$).

$$2906 \text{ cm}^{-1} = 2.906 \times 10^5 \text{ m}^{-1} = \bar{\omega}_0 + \frac{6.626 \times 10^{-34} (+1)}{4(3.14)^2 \rho (3 \times 10^8)}$$

$$\& \quad 2927 \text{ cm}^{-1} = 2.927 \times 10^5 \text{ m}^{-1} = \bar{\omega}_0 + \frac{6.626 \times 10^{-34} (+2)}{4(3.14)^2 \rho (3 \times 10^8)}$$

from these two data, we can calculate the value of two unknowns $\bar{\omega}_0$ & ρ .

$$\bar{\omega}_0 = 2.885 \times 10^5 \text{ m}^{-1}$$

$$\boxed{\bar{\omega}_0 = 2885 \text{ cm}^{-1}}$$

$$\& \quad \boxed{\rho = 2.665 \times 10^{-47} \text{ kg m}^{-2}}$$

$$\rho = \mu r^2$$

$$r = \sqrt{\frac{\rho}{\mu}}$$

$$= 1.292 \times 10^{-10} \text{ m}$$

$$\boxed{r = 0.129 \text{ nm}}$$

$$K = 4\pi^2 \bar{\omega}_0^2 c^2 \mu$$

$$K = ?$$

from.

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chemistry