

2) Average velocity :-

The average velocity is given by the arithmetic mean of the different velocities possessed by the molecules of the gas at a given temperature.

If  $u_1, u_2, u_3, \dots$  are the velocities possessed by  $n_1, n_2, n_3, \dots$  number of molecules respectively, then average velocity will be -

$$\langle U_x \rangle = \frac{u_1 + u_2 + u_3 + \dots + u_N}{N}$$

$$= \frac{1}{N} \sum_{i=1}^N U_{xi}$$

The average velocity may also be defined as the probability that fraction of molecules having velocities between  $U_x$  &  $U_x + dU_x$ , which is represented by -

$$f \langle U_x \rangle dU_x$$

$$\langle U_x \rangle = \int_0^\infty U_x f \langle U_x \rangle dU_x \quad \text{--- ①}$$

where,  $f \langle U_x \rangle dU_x = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} \exp \left( -\frac{MU_x^2}{2RT} \right) U_x^2 dx$

putting this value in eq<sup>s</sup> ① and solving for the integral we get -

$$\langle U_x \rangle = \left( \frac{8RT}{\pi M} \right)^{1/2}$$

Numericals :-

(1). Calculate the average speed of  $\text{CO}_2$  gas at 1684 K temperature?

Hints:-  $\pi = 3.141$

$$R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$(1 \text{ J} = 10^7 \text{ erg})$$

$$M_{\text{CO}_2} = 44 \text{ g mol}^{-1}$$

(2). The average speed at T K temperature of  $\text{N}_2$  gas is  $8 \times 10^5 \text{ cm sec}^{-1}$ . Calculate the temperature (T).

Hints:  $\pi = 3.141$

$$R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$1 \text{ J} = 10^7 \text{ erg}$$

$$M_{\text{N}_2} = 28 \text{ g mol}^{-1}$$

—x—

(3). Root mean square (RMS) velocity :-

The rms velocity is defined as the square root of the mean of the squares of different velocities possessed by molecules of gas at a given temperature.

The rms velocity is given by —

$$\bar{u} = \left( \frac{1}{N} \sum_{i=1}^N u_{xi}^2 \right)^{1/2}$$

This may also be written as —

$$\bar{u} = \int_0^{\infty} u_x^2 f(u_x) du_x$$

where,  $f(u_x) du_x$  represents the fraction of molecules having velocities between  $u_x$  &  $u_x + du_x$ .

where,

$$f(u_x) du_x = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \exp\left(-\frac{Mu_x^2}{2RT}\right) u_x^2 du_x.$$

On substituting this value in integral & solving we get -

$$\bar{u} = \left(\frac{3RT}{M}\right)^{1/2}$$

\* Relationship between Most Probable velocity ( $u_x$ ), Average velocity  $\langle u_x \rangle$  and rms velocity  $\bar{u}$  :-

Since,

$$\text{Most Probable velocity } (u_x) = \left(\frac{2RT}{M}\right)^{1/2}$$

$$\text{Average velocity } \langle u_x \rangle = \left(\frac{8RT}{\pi M}\right)^{1/2}$$

$$\text{and, RMS velocity } \bar{u} = \left(\frac{3RT}{M}\right)^{1/2}$$

Therefore, On comparing them,

$$(u_x) = \langle u_x \rangle = \bar{u}$$

$$\left(\frac{2RT}{M}\right)^{1/2} = \left(\frac{8RT}{\pi M}\right)^{1/2} = \left(\frac{3RT}{M}\right)^{1/2}$$

$$1 = 1.128 = 1.1224$$

Thus,

$$\text{Average velocity } \langle u_x \rangle = 0.9213 \bar{u}$$

$$\& \text{ Most Probable velocity } (u_x) = \sqrt{\frac{2}{3}} \bar{u} = 0.816 \bar{u}.$$

Numericals :-

- ① Calculate the rms velocity of  $O_2$  molecules in the lungs at normal body temperature  $37^\circ C$ .

Hints :-  $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ .

$$1 \text{ J} = 10^7 \text{ erg}$$

$$1 \text{ J} = \text{kg m}^2 \text{ sec}^{-2}$$

$$1 \text{ erg} = \text{g cm}^2 \text{ sec}^{-2}$$

(Ans: -  $4.92 \times 10^4 \text{ cm sec}^{-1}$ .)

- ② Calculate the rms velocity of  $Cl_2$  molecules at  $12^\circ C$  and 78 cm pressure.

Hints :-

$$PV = RT$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Ans: -  $(316.52 \text{ m sec}^{-1})$

- ③ Calculate the temperatures at which the root mean square velocity, the average velocity and the most probable velocity of  $O_2$  gas are all equal to  $1500 \text{ m sec}^{-1}$ .

Hints :-  $(M_{O_2} = 32 \text{ g mol}^{-1})$

Ans: -  $(2886 \text{ K}, 3399 \text{ K} \ \& \ 4330 \text{ K} \text{ respectively.})$

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