*. Distribution of velocities of gases :-

Maxwell distribution law of velocities :-

Gaseous molecules are colloiding with themselves randomly their by changing the molecular velocity.

Let us consider a gas molecule of mass 'm' having a velocity component 'u'. Then, the Kinetic energy 'ε' associated with this velocity component is $\frac{1}{2}mu^2$. The probability that this molecule has its velocity component between u and u+du is given by p(u). Boltzmann had shown that the probability for a molecule to have an energy 'ε' was propotional to $e^{-\varepsilon/KT}$.

$$p(u) \propto e^{-\varepsilon/KT} \propto e^{-mu^2/2KT}$$

$$\text{or} \quad p(u)\,du = A e^{-mu^2/2KT} du \qquad \text{—— (1)}$$

where, 'A' is the constant of proportionality. This constant can be evaluated by requiring that the total probability must be unity. Thus,

$$\int_{-\infty}^{+\infty} p(u)\,du = A\int_{-\infty}^{+\infty} e^{-mu^2/2KT} du = 1 \qquad \text{—— (2)}$$

The range of intigration $+\infty$ to $-\infty$ (velocity component u) has both magnitude and direction. This eq^n — (2) is simplified as-

putting $m/2KT = a$.

$$\int_{-\infty}^{+\infty} e^{-au^2} du = \left(\pi/a\right)^{1/2} = \left(\frac{2\pi KT}{m}\right)^{1/2} \qquad \text{—— (3)}$$

from eq^n — (2) & (3) we have

$$A\left(2\pi KT/m\right)^{1/2} = 1$$

So, $A = (m/2\pi kT)^{1/2}$ — (4)

Now substituting for A in eq^n – (1) we get

$$P(u)\,du = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-mu^2/2kT}\,du \quad \text{— (5)}$$

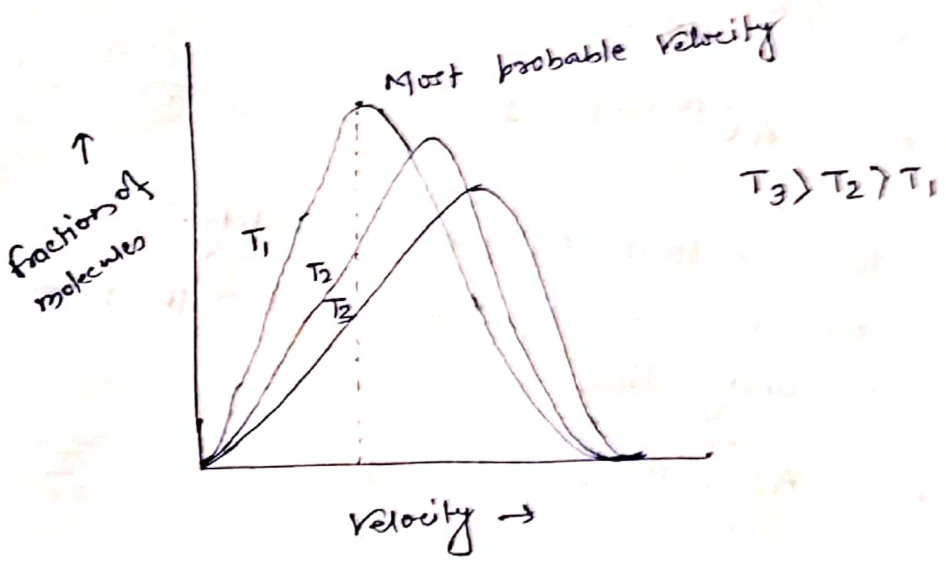
eq^n – (5) is called Maxwell distribution of molecular velocities in one dimension.

In three dimension, the eq^n – (5) is written as —

$$P(c)\,dc = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} c^2 \exp\left(-mc^2/2kT\right) dc \quad \text{— (6)}$$

or $$\frac{dN_c}{N} = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} c^2 \exp\left(-\frac{Mc^2}{2RT}\right) dc \quad \text{— (7)}$$

This may be plotted as:-



At higher temperature, the whole curve shifts to the right. This show that at higher temperature more molecules have higher velocity and fewer molecules have lower velocity.

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1). Most Probable velocity :-

It is defined as the velocity of gas possessed by maximum no. of molecules at a given temperature.

The equation for Maxwell distribution of velocities may be written as —

$$\frac{N}{dU_x} = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} \exp\left(-\frac{MU_x^2}{2RT}\right) U_x^2 \quad \text{— (1)}$$

for maxima & minima conditions, this equation is differentiated w.r.t U_x and equated to zero. Then we get —

$$\left(1 - \frac{MU_x^2}{2RT}\right) U_x \exp\left(-\frac{MU_x^2}{2RT}\right) = 0.$$

since,
$$U_x \exp -\frac{MU_x^2}{2RT} \neq 0.$$

Then,
$$1 - \frac{MU_x^2}{2RT} = 0.$$

or
$$\frac{MU_x^2}{2RT} = 1$$

$$MU_x^2 = 2RT$$

$$\boxed{U_x = \sqrt{\frac{2RT}{M}}}$$

where, 'U_x' is the most probable velocity. M = Mol. mass of the gas