10/03/08

Problems:-

(1) Calculate the energy associated with radiation having wave length 4000 A°.

Sol.- Given $\lambda = 4000\ \mathring{A} = 4000 \times 10^{-10}$ m.

we know that -

$$E = h\nu$$

$$E = \frac{hc}{\lambda}$$

h = Plank's constant = 6.62×10^{-34} J sec

c = Velocity of light = 3×10^{8} m sec^{-1}

Thus-

$$E = \frac{6.62 \times 10^{-34}\ \text{J}\ \cancel{\text{sec}} \times 3 \times 10^{8}\ \cancel{\text{m}}\ \cancel{\text{sec}^{-1}}}{4000 \times 10^{-10}\ \cancel{\text{m}}}$$

$$= \frac{6.62 \times 3 \times 10^{-26}\ \text{J}}{4000 \times 10^{-10}}$$

or, $E = \dfrac{6.62 \times 3 \times 10^{-26}\ J}{4 \times 10^{-7}}$

$= \dfrac{6.62 \times 3 \times 10^{-19}\ J}{4}$

$= \dfrac{19.86 \times 10^{-19}\ J}{4}$

$E = 4.965 \times 10^{-19}\ J\ molecule^{-1}$ $\boxed{E = 5 \times 10^{-19}\ J\ molecule^{-1}}$

or $E = 5 \times 10^{-19} \times 6.023 \times 10^{23}\ J\ mole^{-1}$

$= 5 \times 6.023 \times 10^{4}\ J\ mole^{-1}$

$= 30.115 \times 10^{4}\ J\ mole^{-1}$

$\boxed{E = 300\ KJ\ mole^{-1}}$

2. The wave length range of visible light is 3800 Å – 7600 Å. Calculate the corresponding frequency range in MHz. ?

Ans. Let $\lambda_1 = 3800\ Å$

$= 3800 \times 10^{-10}\ m$

$\lambda_2 = 7600\ Å$

$= 7600 \times 10^{-10}\ m$

$\nu_1 = \dfrac{c}{\lambda_1}$

c = Velocity of light $= 3 \times 10^{8}\ m\ sec^{-1}$

$\nu_1 = \dfrac{3 \times 10^{8}\ m\ sec^{-1}}{3800 \times 10^{-10}\ m}$

$= \dfrac{3 \times 10^{8}\ sec^{-1}}{38 \times 10^{-8}}$

$= \dfrac{3 \times 10^{16}\ sec^{-1}}{38}$

$\therefore \quad \nu_1 = \frac{300 \times 10^{14}\ sec^{-1}}{38}$

$\nu_1 = 7.9 \times 10^{14}\ sec^{-1}$ or Hz.

$\left[\because 10^6\ Hz = 1\ MHz\right]$

$$\boxed{\nu_1 = 7.9 \times 10^{14}\ Hz = 7.9 \times 10^{8}\ MHz.}$$

Similarly, $\nu_2 = \frac{c}{\lambda_2}$

$= \frac{3 \times 10^{8}\ \cancel{m}\ sec^{-1}}{7600 \times 10^{-10}\ \cancel{m}}$

$= \frac{3 \times 10^{8}\ sec^{-1}}{76 \times 10^{-8}}$

$= \frac{3 \times 10^{16}\ sec^{-1}}{76}$

$= \frac{300 \times 10^{14}\ sec^{-1}}{76}$

$= 3.94 \times 10^{14}\ sec^{-1}$ or Hz.

$$\therefore \boxed{\nu_2 = 3.94 \times 10^{14}\ Hz = 3.94 \times 10^{8}\ MHz.}$$

Thus frequency range $= 3.94 \times 10^{8}\ MHz - 7.9 \times 10^{8}\ MHz$

(3) The wave length of radiation with 2.5 μ. Calculate the corresponding wave no.—

Ans. Given $\lambda = 2.5\,\mu = 2.5 \times 10^{-6}\,m = 2.5 \times 10^{-4}\,cm.$

$[1\mu = 10^{-6}\,m]$

we know that —

$$\bar{\nu} = 1/\lambda$$

$$= \frac{1}{2.5 \times 10^{-4}\,cm}$$

$$= \frac{1 \times 10^{4}\,cm^{-1}}{2.5}$$

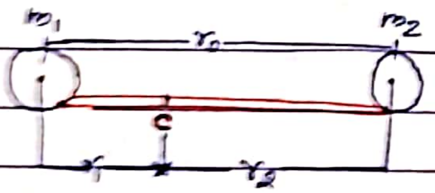
$$= \frac{\cancel{100000}^{\,4000}}{\cancel{25}}\,cm^{-1}$$

$$\boxed{\bar{\nu} = 4 \times 10^{3}\,cm^{-1}}$$

* NOTE :— If we count wave no. ($\bar{\nu}$) in cm^{-1} then —

$$\boxed{\bar{\nu} = \frac{10000}{\mu}}$$

* Rotational Spectroscopy :— [Microwave spectroscopy] :—
* The rigid diatomic molecule :—



Let us consider a diatomic molecule, the atom of which are joined by a rigid bar such as —

$$r_0 = r_1 + r_2 \qquad \text{—— (1)}$$

c is the centre of gravity about which molecule rotates end-over-end rotation. The moment is defined as —

$$m_1 r_1 = m_2 r_2 \quad \text{—— (2)}$$

Therefore —

moment of inertia

$$I = m_1 r_1^2 + m_2 r_2^2$$

then from eqn — (2)

$$I = m_2 r_1 r_2 + m_1 r_1 r_2$$

$$I = r_1 r_2 (m_1 + m_2) \quad \text{—— (3)}$$

from eqn — (2)

$$m_1 r_1 = m_2 r_2$$

$$= m_2 (r_0 - r_1)$$

or $m_1 r_1 = \dfrac{m_2 (r_0 - r_1)}{\cancel{m_T}}$

or $m_1 r_1 = \dfrac{m_2 r_0 - m_2 r_1}{\cancel{m_T}}$

or $m_1 r_1 + m_2 r_1 = m_2 r_0$

or $r_1 (m_1 + m_2) = m_2 r_0$

or $r_1 = \dfrac{m_2 r_0}{m_1 + m_2}$

Similarly,

$r_2 = \dfrac{m_1 r_0}{m_1 + m_2}$ —— (4)

Now, putting the value of r_1 & r_2 in eqn -(3) we get —

$$I = \frac{m_2 r_0}{m_1+m_2} \times \frac{m_1 r_0}{m_1+m_2} (m_1+m_2)$$

$$= \frac{m_2 r_0 \cdot m_1 r_0}{(m_1+m_2)^{\cancel{2}}} \cancel{(m_1+m_2)}$$

$$I = \frac{m_1 m_2}{m_1+m_2} \cdot r_0^2$$

$$\boxed{I = \mu r_0^2}$$

where, μ = reduced mass $= \frac{m_1 m_2}{m_1+m_2}$

On solving Schrödinger eqn. the allowed rotational energy levels for diatomic molecule is —

$$\boxed{E_J = \frac{h^2}{8\pi^2 I} J(J+1) \quad \text{Joules.}}$$

where, J = Rotational quantum no.
= 0, 1, 2, 3, ...

Since,

$$E = h\nu$$

$$E = \frac{hc}{\lambda}$$

$$E = hc\bar{\nu} \qquad \left[\because \bar{\nu} = 1/\lambda\right]$$

$$\bar{\nu} = \frac{E_J}{hc} = \varepsilon_j$$

$$\bar{\nu} = \varepsilon_j = \frac{h^{\cancel{2}}}{8\pi^2 I \cancel{h} c} J(J+1) \quad cm^{-1}$$

$$\bar{\nu} = \boxed{\varepsilon_j = \frac{h}{8\pi^2 I c} J(J+1) \quad cm^{-1}}$$

$$\Rightarrow \boxed{E_j = B\, J(J+1)\ cm^{-1}}$$

where, B = Rotational Constant $= \dfrac{h}{8\pi^2 I c}$.

Various Conditions:-

if $J = 0$

$E_j = B \times 0(0+1)\ cm^{-1}$

$E_0 = 0$

if $J = 1$.

$E_j = B \times 1(1+1)\ cm^{-1}$

$E_1 = 2B\ cm^{-1}$

if $J = 2$

$E_j = B \times 2(2+1)\ cm^{-1}$

$E_2 = 6B\ cm^{-1}$

if $J = 3$

$E_j = B \times 3(3+1)\ cm^{-1}$

$E_3 = 12B\ cm^{-1}$

if $J = 4$

$E_j = B \times 4(4+1)\ cm^{-1}$

$E_4 = 20B\ cm^{-1}$

if $J = 5$

$E_j = B \times 5(5+1)\ cm^{-1}$

$E_5 \quad 30B\ cm^{-1}$

if $J = 6$.

$E_j = B \times 6(6+1)\ cm^{-1}$

$E_6 = 42B\ cm^{-1}$

if $J = 7$

$E_j = B \times 7(7+1)\ cm^{-1}$

$E_7 = 56B\ cm^{-1}$