## * Commutation of Operators :-

If two numbers (symbol) are $a$ & $b$ then -

$$a \times b = b \times a$$

However, If $\hat{A}$ & $\hat{B}$ are two operators, then their product may or may not be equal.

i.e.

$$\hat{A} \times \hat{B} = \hat{B} \times \hat{A}$$

or

$$\hat{A} \times \hat{B} \neq \hat{B} \times \hat{A}$$

When, $\hat{A} \times \hat{B} = \hat{B} \times \hat{A}$ then this is called Commution of operator (Commutative operators).

If $\hat{A} \times \hat{B} = \hat{B} \times \hat{A}$

then,

$$[\hat{A} \times \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = 0.$$

- operators can be Complex. They can also be vector.

**Q.** Show that the commutator $\left[x, \frac{d}{dx}\right] = -1$

**Soln.**

$$\left[x, \frac{d}{dx}\right] = x\frac{d}{dx} - \frac{d}{dx}x$$

Let $\psi(x)$ be the operand, then operating on $\psi(x)$ by the R.H.S expression, we have -

$$\left(x\frac{d}{dx} - \frac{d}{dx}x\right)\psi = x\frac{d}{dx} - \frac{d}{dx}(x\psi) = x\frac{d\psi}{dx} - x\frac{d\psi}{dx} - \psi\frac{dx}{dx}$$

$$= -\psi$$

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or $\left(x\frac{d}{dx}-\frac{d}{dx}x\right)\psi = -\psi$.

or $\left(x\frac{d}{dx}-\frac{d}{dx}x\right) = -1$

or $\left[x\ \frac{d}{dx}\right] = -1$.

# Linear operators :-

An operator $\hat{A}$ is said to be linear when it satisfies the following relations -

$$\hat{A}\left[c_1 f_1(x) + c_2 f_2(x)\right] = c_1 \hat{A} f_1(x) + c_2 \hat{A} f_2(x)$$

where, $c_1$ & $c_2$ are two real or complex constants and $f_1(x)$ & $f_2(x)$ are functions of $x$.

Q. Show that -

1). $[\hat{A}\ \hat{B}] = -[\hat{B}\ \hat{A}]$

Sol$^n$ $[\hat{A}\ \hat{B}] = [\hat{A}\hat{B} - \hat{B}\hat{A}]$

$= -[\hat{B}\hat{A} - \hat{A}\hat{B}]$

$= -[\hat{B}\hat{A}]$

ii) $[\hat{A}^2 \hat{B}] = \hat{A}[\hat{A}\hat{B}] + [\hat{A}\hat{B}]\hat{A}$

Sol<sup></sup>

Soln from R.H.S.

$$\hat{A}[\hat{A}\hat{B}] + [\hat{A}\hat{B}]\hat{A} = \hat{A}[\hat{A}\hat{B} - \hat{B}\hat{A}] + [\hat{A}\hat{B} - \hat{B}\hat{A}]\hat{A}$$

$$= \hat{A}^2\hat{B} - \cancel{\hat{A}\hat{B}\hat{A}} + \cancel{\hat{A}\hat{B}\hat{A}} - \hat{B}\hat{A}^2$$

$$= \hat{A}^2\hat{B} - \hat{B}\hat{A}^2$$

$$= [\hat{A}^2\hat{B}] \quad \text{L.H.S}$$

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