Hermitian Operator :-

Any physically observable quantity can be expressed in quantum mechanics by a Hermitian operator. An operator is said to be Hermitian if the turn over rule is obeyed.

A Hermitian is a linear operator $\hat{A}$ that satisfied the following condition :-

$$\int \psi_1^* \hat{A}(\psi_2)\, d\tau = \int \psi_1 \hat{A}(\psi_2^*)\, d\tau$$

Where, ψ_1 & ψ_2 are the two wave functions.

ψ_1^* & ψ_2^* are complex conjugate of two wave function ψ_1 & ψ_2 respectively.

$d\tau$ is the small volume element.

* Theorem of operators :-

Properties of Hermitian operator :-

There are two very important characteristics of a Hermitian operator.

(1). The eigen values are real i.e. positive or negative

Let $\hat{A}$ be the Hermitian operator, ψ its eigen function and λ the eigen value then,

$$\hat{A}\psi = \lambda\psi \quad \text{——— (1)}$$

also, $(\hat{A}\psi)^* = \lambda^* \psi^*$ ——— (2)

on multiplying eq^n – (1) by ψ^* and eq^n – (2) by ψ and then integrating —

$$\int \psi^* \hat{A} \psi \, d\tau = \int \psi^* \lambda \psi \, d\tau = \lambda \int \psi^* \psi \, d\tau$$

$$\& \quad \int \psi (\hat{A}\psi)^* \, d\tau = \int \psi \lambda^* \psi^* d\tau = \lambda^* \int \psi \psi^* d\tau$$

Since, $\hat{A}$ is Hermitian then

$$\int \psi^* (\hat{A}\psi) \, d\tau = \int \psi (\hat{A}\psi)^* \, d\tau$$

$$\therefore \quad \lambda \int \psi^* \psi \, d\tau = \lambda^* \int \psi \psi^* \, d\tau$$

$$\lambda = \lambda^*$$

i.e. λ is real.

(2). Eigen functions corresponding to different eigen values are orthogonal to each other :—

Let φ_1 & φ_2 are two eigen functions of a Hermitian operator $\hat{A}$ corresponding to two eigen values λ_1 & λ_2 respectively.

The orthogonality condition is –

$$\int \varphi_1 \varphi_2 \, d\tau = 0$$

or $\quad \int \varphi_1 \varphi_2^* \, d\tau = 0$

or $\quad \int \varphi_1^* \varphi_2 \, d\tau = 0$

The eigen value equation are —

$$\hat{A}\,\psi_1 = \lambda_1 \psi_1 \qquad \text{——— (1)}$$

and $$\hat{A}\,\psi_2 = \lambda_2 \psi_2 \qquad \text{——— (2)}$$

on multiplying eqⁿ — (1) by ψ_2^* & intigrating —

$$\int \psi_2^* \hat{A}\,\psi_1 \, d\tau = \int \psi_2^* \lambda_1 \psi_1 \, d\tau = \lambda_1 \int \psi_2^* \psi_1 \, d\tau$$

Since, $\hat{A}$ is Hermition then —

$$\int \psi_2^* \hat{A}\,\psi_1 d\tau = \int \psi_1 (\hat{A}\,\psi_2)^* d\tau = \int \psi (\lambda_2 \psi_2)^* d\tau$$

$$= \lambda_2^* \int \psi_1 \psi_2^* \, d\tau$$

$$= \lambda_2 \int \psi_1 \psi_2^* \, d\tau$$

Thus, $$\lambda_1 \int \psi_2^* \psi_1 \, d\tau = \lambda_2 \int \psi_1 \psi_2^* . d\tau$$

or $$\lambda_1 - \lambda_2 \int \psi_1 \psi_2^* \, d\tau = 0$$

But $\lambda_1 \neq \lambda_2$

Thus, $\int \psi_1 \psi_2^* \, d\tau = 0$.

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