2) Average velocity :-

The average velocity is given by the arithmatic mean of the different velocities possessed by the molecules of the gas at a given temperature.

If $u_1, u_2, u_3 \ldots\ldots$ are the velocities possessed by $n_1, n_2, n_3 \ldots\ldots$ number of molecules respectively, then average velocity will be –

$$\langle U_x \rangle = \frac{u_1 + u_2 + u_3 + \ldots\ldots U_N}{N}$$

$$= \frac{1}{N} \sum_{i=1} U_{xi}$$

The average velocity may also be defined as the probability that fraction of molecules having velocities between U_x & $U_x + dU_x$, which is represented by –

$$f\langle U_x \rangle dU_x$$

or
$$\langle U_x \rangle = \int_0^\infty U_x f\langle U_x \rangle dU_x \quad \text{--- (1)}$$

where,
$$f\langle U_x \rangle dU_x = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \exp\left(-\frac{MU_x^2}{2RT}\right) U_x^2 dx$$

putting this value in eq^n (1) and solving for the intigral we get –

$$\langle U_x \rangle = \left(\frac{8RT}{\pi M}\right)^{1/2}$$

Numericals :-

(1). Calculate the average speed of CO_2 gas at 1684 K temperature?

Hints:- $\pi = 3.141$

$R = 8.314\ J\ K^{-1}\ mol^{-1}$

$(1J = 10^7\ erg)$

$M_{CO_2} = 44\ g\ mol^{-1}$.

(2). The average speed at T K temperature of N_2 gas is $8 \times 10^5\ cm\ sec^{-1}$. Calculate the temperature (T).

Hints: $\pi = 3.141$

$R = 8.314\ J\ K^{-1}\ mol^{-1}$

$1J = 10^7\ erg$

$M_{N_2} = 28\ g\ mol^{-1}$.

—x—

(3). Root mean square (RMS) velocity :-

The rms velocity is defined as the square root of the mean of the squares of different velocities possessed by molecules of gas at a given temperature.

The rms velocity is given by-

$$\bar{u} = \left(\frac{1}{N} \sum_{i=1}^{N} U_{x_i}^2 \right)^{1/2}$$

This may also be written as —

$$\bar{u} = \int_0^\infty U_x^2\, f\langle U_x \rangle\, dU_x$$

where, $f\langle U_x \rangle\, dU_x$ represents the fraction of molecules having velocities between U_x & $U_x + dU_x$.

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where,

$$f\langle U_x \rangle \, dU_x = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \exp\left(-\frac{MU_x^2}{2RT}\right) U_x^2 \, dU_x .$$

On substituting this value in indigral & solving we get –

$$\bar{u} = \left(\frac{3RT}{M}\right)^{1/2} .$$

* <u>Relationship between Most Probable velocity (U_x), Average velocity $\langle U_x \rangle$ and rms velocity $\bar{u}$:–</u>

Since,

Most Probable velocity $(U_x) = \left(\frac{2RT}{M}\right)^{1/2}$

Average velocity $\langle U_x \rangle = \left(\frac{8RT}{\pi M}\right)^{1/2}$

and. RMS velocity $\bar{u} = \left(\frac{3RT}{M}\right)^{1/2}$

Therefore, on comparing them,

$$(U_x) = \langle U_x \rangle = \bar{u} .$$

$$\left(\frac{2RT}{M}\right)^{1/2} = \left(\frac{8RT}{\pi M}\right)^{1/2} = \left(\frac{3RT}{M}\right)^{1/2}$$

$$1 = 1.128 = 1.1224$$

Thus,

Average velocity $\langle U_x \rangle = 0.9213 \times \bar{u}$

& Most Probable velocity $(U_x) = \sqrt{2/3}\ \bar{u} = 0.816\ \bar{u}$.

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Numericals :-

(1) Calculate the rms velocity of O_2 molecules in the lungs at normal body temperature $37°c$.

Hints :- $R = 8.314\ J\ K^{-1}\ mol^{-1}$.

$1J = 10^7\ erg$

$1J = kg\ m^2\ sec^{-2}$

$1\ erg = g\ cm^2\ sec^{-2}$.

(Ans: - $4.92 \times 10^4\ cm\ sec^{-1}$.)

(2) Calculate the rms velocity of Cl_2 molecules at $12°c$ and 78 cm pressure.

Hints :-

$PV = RT$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Ans:- ($316.52\ m\ sec^{-1}$)

(3) Calculate the temperatures at which the root mean square velocity, the average velocity and the most probable velocity of O_2 gas are all equal to $1500\ m\ sec^{-1}$.

Hints:- ($M_{O_2} = 32\ g\ mol^{-1}$)

Ans:- (2886 K, 3399 K & 4330 K respectively.)

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