

# The Binomial Probability Distribution

If chances of occurrence of an event in an experiment is  $P$  and no chance to occur that event is  $q$ , then in  $n$  independent experiments,  $x$  chances of occurrence of an event is

$$P(x^n) = {}^n C_x p^x q^{n-x}$$

Probability distribution given by above equation is called Binomial distribution.

Value of  $x$  in above equation is  $0, 1, 2, 3, \dots, n$  then

$$P(1) = \text{chances of one success of probability} \\ = {}^n C_1 p^1 q^{n-1}$$

$$P(2) = \text{chances of two success of probability} \\ = {}^n C_2 p^2 q^{n-2}$$

$$P(3) = \text{chances of three success probability} \\ = {}^n C_3 p^3 q^{n-3}$$

$$P(x) = \text{chances of } x \text{ success probability} \\ = {}^n C_x p^x q^{n-x} \text{ etc.}$$

It means these probability have been taken by the expression of  $(q+p)^n$  in serial steps.

$$(q+p)^n = q^n + {}^n C_1 p q^{n-1} + {}^n C_2 p^2 q^{n-2} + \dots + {}^n C_n p^n$$

If  $N$  set is there and in each set there is  $n$  attempt then relative



frequency can be given by following steps of expansion.

$$N(q+p)^n = N[q^n + {}^n C_1 p q^{n-1} + {}^n C_2 p^2 q^{n-2} + \dots + n p^{n-1} q + p^n]$$

where 0, 1, 2, 3, ..., n success is as per set no is  $Nq^n$ ,  $N \cdot {}^n C_1 p q^{n-1}$ ,  $N \cdot {}^n C_2 p^2 q^{n-2}$ , ...,  $N \cdot {}^n C_n p^n$ .

Example: In following table  $x$  = no. of occurrence of event and  $f$  = relative frequency. We have to detect whether distribution is Binomial or not and what is its mean and standard deviation ( $\sigma$ ).

$x$	0	1	2	3	4	5	6
$f$	729	1458	1215	540	135	18	1
	0	1458	2430	1620	540	90	6

$$\sum x f = 6144$$

$$\sum f = 4096$$

Calculation. Here  $q/6 \propto 729$  and  $q/6 \propto 1$

$$\therefore \frac{p}{q} = \frac{1}{(729)^{1/6}} = \frac{1}{3} \Rightarrow 3p = q = 1-p$$



$$\Rightarrow 4p = 1$$

$$\therefore p = \frac{1}{4} \text{ and } q = \frac{3}{4}$$

This indicates that the distribution is binomial in which

$$p = \frac{1}{4} \text{ and } q = \frac{3}{4} \text{ and } n = 6$$

Therefore mean =  $np = 6 \times \frac{1}{4} = \frac{3}{2}$   
and standard deviation

$$= \sqrt{npq}$$

$$= \sqrt{6 \times \frac{1}{4} \times \frac{3}{4}}$$

$$= \frac{3}{2\sqrt{2}} = \frac{3}{2 \times 1.414} = \frac{3}{2.828}$$

$$= 1.06$$

Mean could also be deduced by following formula

$$\frac{\sum xf}{\sum f} = \frac{6144}{4096} = \frac{3}{2} = 1.5 \text{ Ans}$$