Find the equation of the tangent at any point (x_1, y_1) to the parabola $y^2 = 4ax$.

Solution

The equation of the parabola is

$$y^2 = 4ax$$

Let $f(x, y) = y^2 - 4ax$

$\therefore \frac{\partial f}{\partial x} = -4a$ and $\frac{\partial f}{\partial y} = 2y$

$$\therefore \frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{-4a}{2y} = \frac{2a}{y}$$

at the point (x_1, y_1) $\left(\frac{dy}{dx}\right)_{x,y} = \frac{2a}{y_1}$

$\left(\frac{dy}{dx}\right)_{x,y} = \frac{2a}{y_1}$, which is the slope of tangent at (x_1, y_1)

Hence the equation of tangent at the point (x_1, y_1) to the parabola is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x,y)} (x - x_1)$$

$$y - y_1 = \frac{2a}{y_1}(x - x_1)$$

$$yy_1 - y_1^2 = 2ax - 2ax_1 \quad \text{--- (2)}$$

But the point (x_1, y_1) lies on the parabola $y^2 = 4ax$ also, then

$$y_1^2 = 4ax_1$$

Hence from (2) the required equation of the tangent at (x_1, y_1) to the parabola

$$yy_1 - 4ax_1 = 2ax - 2ax_1$$

or, $yy_1 = 2ax + 2ax_1$

$$yy_1 = 2a(x + x_1)$$

$\Rightarrow$ which is the required equation of the tangent.

Find the equation of the tangent at the point $(at^2, 2at)$ to the parabola $y^2 = 4ax$

Solution The equation of parabola is

$$y^2 = 4ax$$

Let $F(x, y) = y^2 - 4ax$

$\therefore \frac{\partial F}{\partial x} = -4a$ and $\frac{\partial F}{\partial y} = 2y$

$$\therefore \left(\frac{dy}{dx}\right) = \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{\cancel{4}^{2}a}{\cancel{2}y}$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

$\therefore \frac{dy}{dx}$ at the point $(at^2, 2at) = \frac{2a}{2at}$

$$= \frac{1}{t}$$

which is the slope of the tangent at the point $(at^2, 2at)$

$\therefore$ The equation of the tangent at the point $(at^2, 2at)$ to the parabola is

(47)

(1)

$$y - 2at = \left(\frac{dy}{dx}\right)_{(at^2, 2at)} (x - at^2)$$

$$\Rightarrow y - 2at = \frac{1}{t}(x - at^2)$$

$$\Rightarrow yt - 2at^2 = x \cancel{+} - at^2$$

$$\Rightarrow yt = x - at^2 + 2at^2$$

$$yt = x + at^2$$

which is the required equation