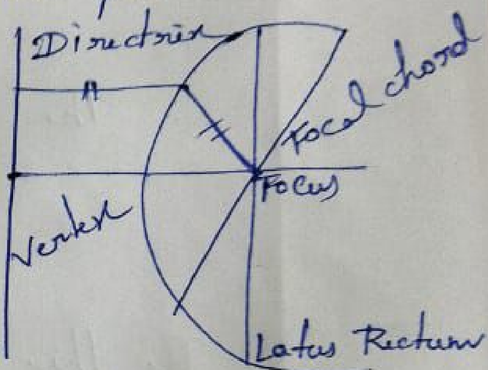
STANDARD EQUATION OF PARABOLA ELLIPSE AND HYPERBOLA

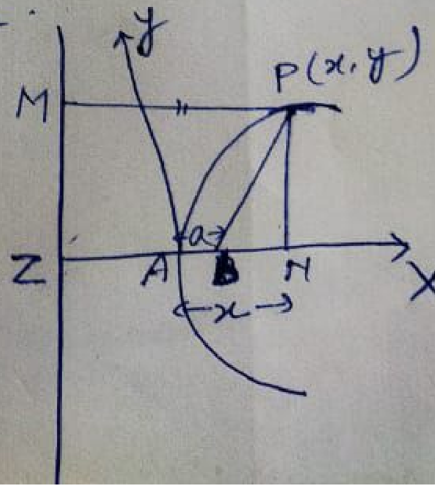
PARABOLA

Definition of parabola:- A parabola is the locus of a point which moves in a plane in such a way that its distance from a fixed point in the plane is always equal to its distance from a fixed straight line in the same plane.



The fixed point is called the focus of parabola and the fixed straight line is called the directrix of the parabola. The line through the focus and perpendicular of the directrix is called the axis of the parabola. The mid point of this axis between focus and the directrix is called the vertex of the parabola.

Equation of the parabola:- Let b be focus of the parabola and ZM be the directrix of the parabola.



From b we draw a perpendicular bz to the directrix.

Let A be mid point of bz.

$\therefore$ Ab = Az, A lies

on parabola.
Let A be origin and Ax, Ay the co-ordinate axis.
Let P(x, y) be a point on the parabola. We draw perpendicular from P on the directrix and Ax be PM and PN. respectively.

$\therefore AN = x$ and $PN = y$

We join P and b.

Let $bz = 2a$

$\therefore Ab = Az = a$

$\therefore$ The co-ordinates of b are $(a, 0)$

$$\therefore Pb^2 = (x-a)^2 + (y-0)^2$$
$$= (x-a)^2 + y^2 \quad \text{---} \quad (i)$$

By the definition of parabola

$$Pb = PM$$
$$= ZN$$
$$= Az + AN$$

or, $Pb = a + x$

$$\therefore Pb^2 = (x+a)^2 \quad \text{---} \quad (ii)$$

From (i) and (ii) we have

$$(x-a)^2 + y^2 = (x+a)^2$$

or, $\cancel{x^2} - 2ax + \cancel{a^2} + y^2 = \cancel{x^2} + 2ax + \cancel{a^2}$

or, $y^2 = 2ax + 2ax$

or, $y^2 = 4ax$

$$\boxed{\therefore y^2 = 4ax}$$

which is the standard equation of parabola.

⇒ Find the Condition that the line $y = mx + c$ may touch the parabola $y^2 = 4ax$.

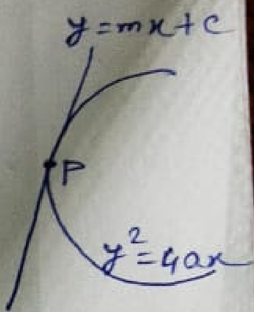
Solution: The equation of parabola is

$$y^2 = 4ax \quad \text{--- (1)}$$

The equation of straight line is

$$y = mx + c \quad \text{--- (2)}$$

The point of intersection of the parabola and the straight line is obtained by solving (1) and (2). Putting the value of y from (2) in (1) we get



$$(mx + c)^2 = 4ax$$

$$\text{or, } m^2x^2 + 2mcx + c^2 = 4ax$$

$$m^2x^2 + 2(mc - 2a)x + c^2 = 0 \quad \text{--- (2)}$$

If the line (2) touches the parabola (1) then the two value of equation (3) will be same. The Condition for which is

$$\{2(mc - 2a)\}^2 - 4m^2c^2 = 0 \quad [\because b^2 = 4ac = 0]$$

$$\text{or, } 4(m^2c^2 - 4mca + 4a^2) = 4m^2c^2$$

$$\text{or, } m^2c^2 - 4mca + 4a^2 - m^2c^2 = 0$$

$$4a^2 = 4mca$$

$$a = mc$$

$$\therefore c = \frac{a}{m} \text{ which is required Condition.}$$