Analytical geometry of three dimensions

(1)

Q.) **The plane :-**

Find the intercept form of the equation of a plane

OR

Find the equation of a plane which makes intercepts a, b and c from x, y and z axes respectively.

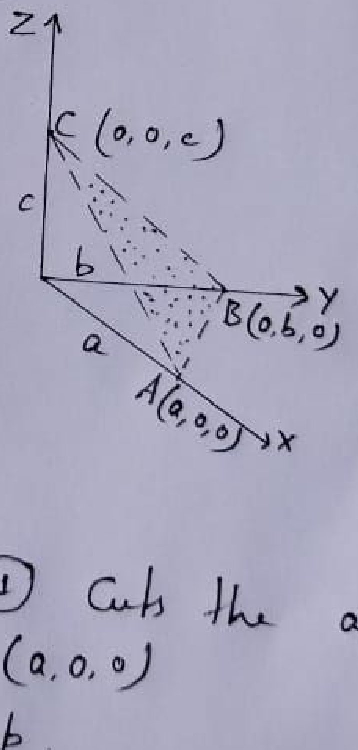
OR

Find the equation of the plane in the form of

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 .$$

Ans:- Let the equation of the plane be

$$lx + my + nz = p \quad \text{—(1)}$$



Let a, b, c be the parts of the axes of x, y, z intercepted between the origin and the plane.

Now the plane (1) cuts the axis of x in the point A(a, 0, 0)

$$\therefore la + m\cdot 0 + n\cdot 0 = p.$$

$$\text{i.e. } l = p/a .$$

Also, the plane (1) cuts the y and the z axes at the points B(0, b, 0) and C(0, 0, c) respectively.

$\therefore \; l\cdot 0 + m\cdot b + n\cdot 0 = p$, i.e. $m = p/b$

and $l\cdot 0 + m\cdot 0 + n\cdot c = p$, i.e. $n = p/c$

substituting the values of l, m and n in ①, we get

$$\frac{p}{a}x + \frac{p}{b}y + \frac{p}{c}z = p.$$

Hence $$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

This is the required equation of the plane.

Q.) Find the equation of the plane passing through the intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and the point $(1,1,1)$

Sol:- Any plane through the intersection of the given planes is

$$x+y+z-6 + k(2x+3y+4z+5) = 0$$

If it passes through $(1,1,1)$ then

$$1+1+1-6+k(2+3+4+5)=0$$

$$\therefore \; k = \frac{3}{14}$$

Hence the required equation of the plane is

$$x+y+z-6+\frac{3}{14}(2x+3y+4z+5)=0$$

or, $14(x+y+z-6)+3(2x+3y+4z+5)=0$

or, $20x+23y+26z-69=0$