Similarly

$$d^*_{0k0} = k\, d^*_{010}, \quad d^*_{00l} = l\, d^*_{001}$$

Here $d^*_{100}$, $d^*_{010}$, $d^*_{001}$ are the repeat distances along $a^*$, $b^*$ and $c^*$ respectively.

If $a^*$, $b^*$, $c^*$ be taken as the repeat distances (equal to $d^*_{100}$, $d^*_{010}$, $d^*_{001}$) then the distance of any lattice point along $a^*$ from origin of the reciprocal lattice is written as

$$r^*_{h00} = h a^*$$

Similarly, the lattice points along $b^*$ and $c^*$ can be written as

$$r^*_{0k0} = k b^*$$

and $$r^*_{00l} = l c^*$$

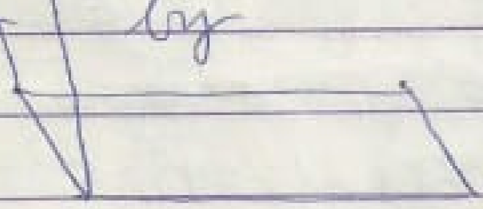
Thus any lattice point in the reciprocal lattice is given by

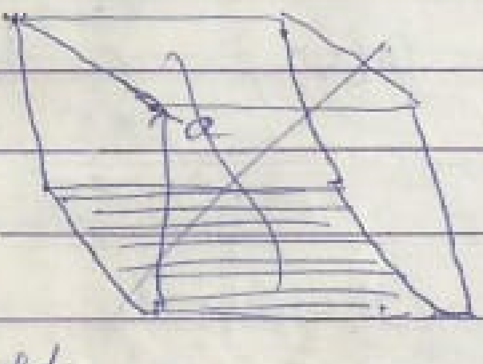
$$\vec{r}^*_{hkl} = h\vec{a}^* + k\vec{b}^* + l\vec{c}^*$$

<u>Vector ~~de~~development of the reciprocal lattice:</u>

Let us consider a unit cell having unit cell dimension as a, b and c as shown in the figure. In the figure $d_{100}$ is the height of the unit cell.

The volume of the unit cell is given by

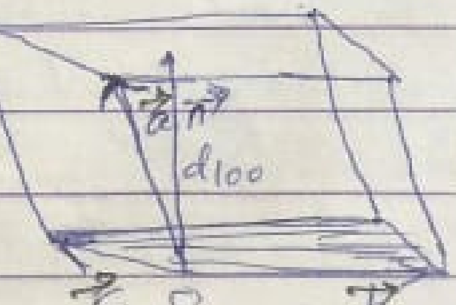





$$V = \text{area of base} \times \text{height}$$

or $$V = \text{area} \times d_{100}$$

or $$\frac{1}{d_{100}} = \frac{\text{area}}{V} \quad \text{---(1)}$$




~~If $\sigma_{hkl}$ be the reciprocal of $d_{hkl}$ then $\sigma_{hkl} = \frac{1}{d_{hkl}}$~~

If $\vec{n}$ be the unit vector of the normal to the plane bc, then $\vec{d}^*_{100} = \frac{1}{d_{100}}\vec{n}$

~~$\vec{\sigma}_{hkl} = \frac{1}{d_{hkl}}\vec{n}$~~ $\therefore$ ~~$\vec{\sigma}_{100} = \frac{1}{d_{100}}\vec{n}$~~

since $d^*_{100} = 1/d_{100}$

Lattice planes and Miller Indices:

A crystal is made up of a large number of parallel equidistant planes passing through lattice points called lattice planes. The directions of these planes are specified by indices h, k, l called Miller indices. These indices are defined as the reciprocals of the intercepts made by the plane on the three crystallographic ~~Rectangular~~ axes.

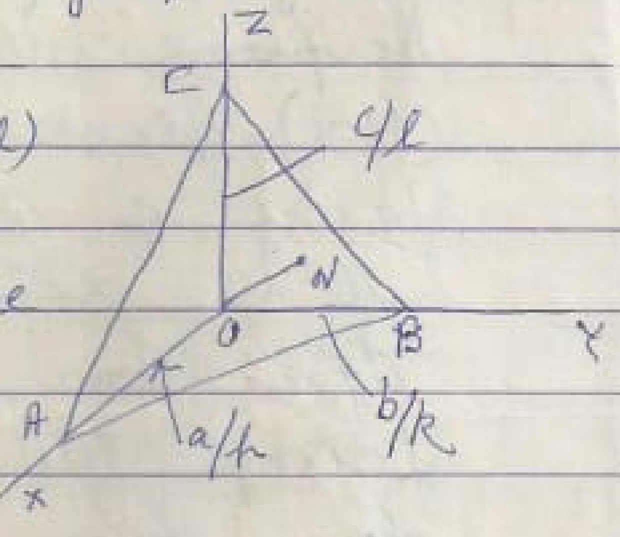
The Miller indices hkl for any plane cutting intercepts 2, 3 and 6 units along three crystalographic axes can be obtained in the following way.

| | |
|---|---|
| 1. Obtain the intercepts on the three crystallographic axes OX, OY, OZ: They are | 2 , 3 , 6 |
| 2. obtain their reciprocals | $\frac{1}{2}$ , $\frac{1}{3}$ , $\frac{1}{6}$ |
| 3. Clear the fraction (multiplying the reciprocals by LCM of their denominators) | 3 , 2 , 1 |

Hence, the Miller indices of the plane will be 321 and they ~~at~~ are together written as (321).

Interplaner spacing of lattice planes:

To obtain the interplaner spacing of a set of parallel lattice planes we shall consider a simple unit cell in which the crystallographic axes are ~~at~~ orthogonal.




Let a plane ABC ~~p~~ has (hkl) as Miller indices. The reference plane from which the spacing of ABC plane is to be obtained passes through the origin. The reference plane and the ABC plane are parallel and the spacing is denoted as $d_{hkl}$. Let ON be the normal drawn from the origin O to ABC plane. Obviously the intercepts of the plane ABC on the crystallographic axes are $a/h$, $b/k$, $c/l$ on OX, OY, OZ respectively. If $\alpha, \beta, \gamma$ be the angles made by ON with OX, OY and OZ, then

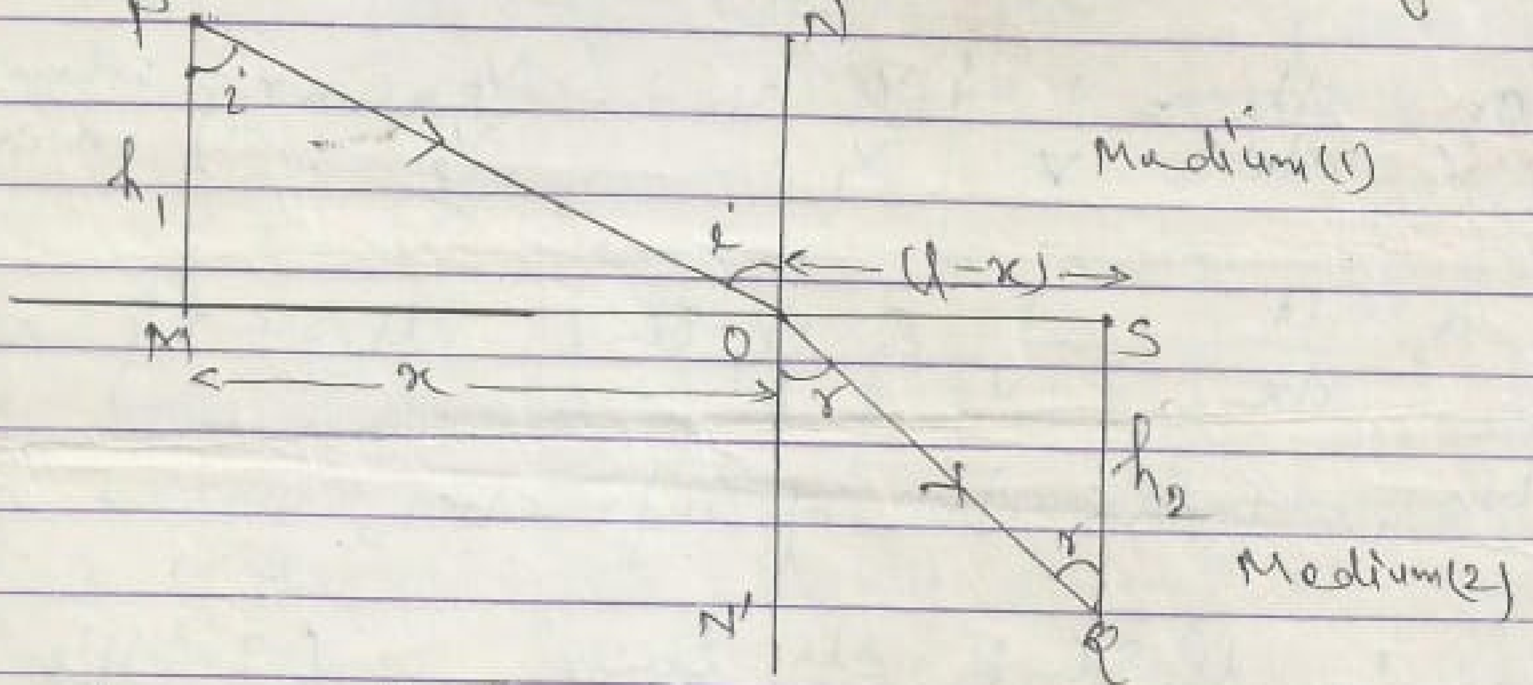
$$ON = \frac{a}{h}\cos\alpha = \frac{b}{k}\cos\beta = \frac{c}{l}\cos\gamma$$

i.e. angle of incidence is equal to the angle of reflection. This is second law of reflection.

Further from Fermat's principle, the path POQ should be minimum. This is only possible when the plane containing POQ must be normal to mirror MR. Since this plane contains normal ON also, incident ray (PO), normal (ON) and reflected ray (OQ) all are co-planer. This is first law of reflection.

Thus reflection is explained by Fermat's principle.

## Fermat's principle from the laws of refraction:




Let a light ray from a point P in medium(1) of refractive index $\mu_1$ be refracted to another point Q in medium(2) of refractive index $\mu_2$ along the path POQ. Let i and r be the angle of incidence and angle of refraction respectively and $h_1$ and $h_2$ be the length of the perpendiculars drawn from P and Q to the refracting surface.

Let MS = l and MO = x, So, OS = (l-x)

If the time taken by the ray of light to go from P to Q via the point O is t, then,

$$t = \frac{PO}{V_1} + \frac{OQ}{V_2}$$

where $V_1$ and $V_2$ are the velocities of light in the two media respectively. Putting the values of PO and OQ from figure, we get

Time taken by the light ray ~~in~~ going from P to Q via O is,
$$t = \frac{PO}{c} + \frac{OQ}{c} \quad \text{where } c = \text{velocity of the light}$$
$$\therefore t = \frac{\sqrt{h_1^2 + x^2}}{c} + \frac{\sqrt{h_2^2 + (l-x)^2}}{c} \quad \text{--- (1)}$$

The length of path POQ traversed by light and the time 't' taken for this purpose will depend on the point of incidence O i.e. on the value of $x$. Differentiating eqn (1) with respect to $x$, we get
$$\frac{dt}{dx} = \frac{\frac{1}{2}(h_1^2 + x^2)^{-1/2} \cdot 2x}{c} + \frac{\frac{1}{2}[h_2^2 + (l-x)^2]^{-\frac{1}{2}} 2 \cdot (l-x)(-1)}{c}$$
$$\text{or, } \frac{dt}{dx} = \frac{x}{c\sqrt{h_1^2 + x^2}} - \frac{(l-x)}{c\sqrt{h_2^2 + (l-x)^2}}$$
$$\text{or, } \frac{dt}{dx} = \frac{\sin i}{c} - \frac{\sin r}{c} \quad \text{--- (2)}$$

$$\because \sin i = \frac{x}{\sqrt{h_1^2 + x^2}}$$
$$\sin r = \frac{l-x}{\sqrt{h_2^2 + (l-x)^2}}$$

If the ray goes from P to Q along path POQ by obeying laws of reflection, then
$$i = r$$
$$\therefore \sin i = \sin r$$
Hence by eqn (2) $\frac{dt}{dx} = 0$;

$t = 0$ const.

Thus the laws of reflection are taken to be true, then the time (t) taken (t) by light to go from one point P to another point Q, by reflection will be stationary i.e. either maximum or minimum. This proves Fermat's principle.

+ not change.

Law of reflection from Fermat's principle:

According to Fermat's principle, the time taken (t) by light in travelling the path POQ should be stationary. Hence $\frac{dt}{dx} = 0$

Hence, eqn (2) becomes $\frac{\sin i}{c} - \frac{\sin r}{c} = 0$
$$\text{or, } \sin i = \sin r$$
$$\text{or, } i = r$$